

Charged Clusters in Liquid Helium

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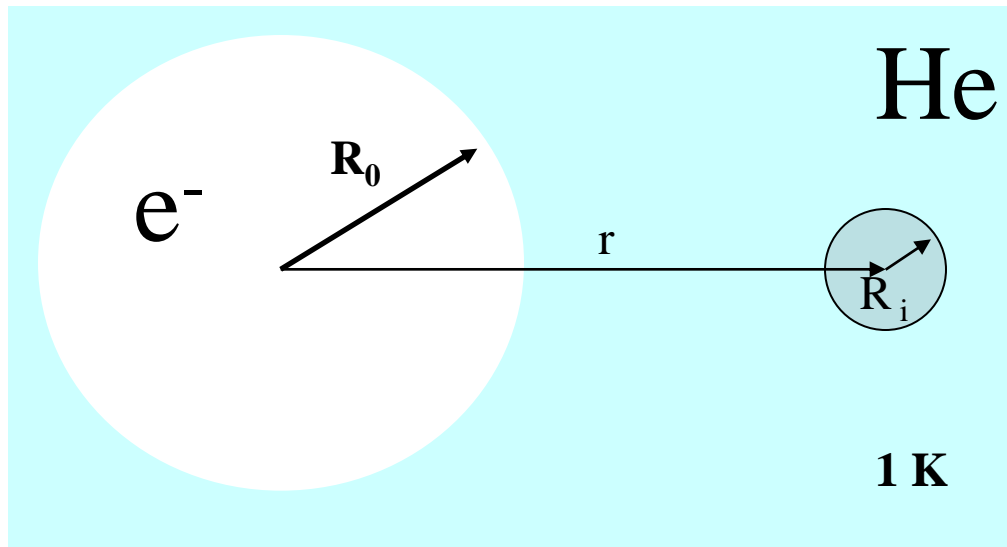
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Ne , Ar, Kr, Xe, H₂, N₂



**E. V. Lebedeva, A.M. Dyugaev, and P. D. Grigor'ev,
Bound States of an Electron and a Macroscopic Cluster at a Helium Surface, JETP,
98, 2, 441-444, (2004).**

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Various charged complexes at the surface of liquid helium, HAIT Journal of
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**Grigoriev P.D., Dyugaev A.M., and Lebedeva E.V.
Temperature Dependence of the Spectrum of Electrons Levitating above Solid
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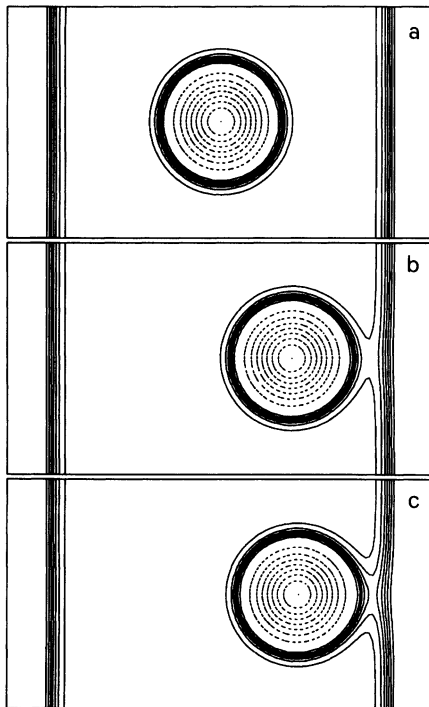
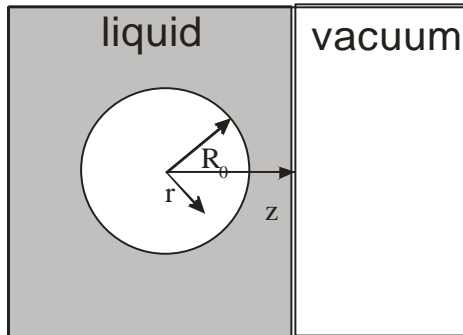
**A.M. Dyugaev, P. D. Grigor'ev and E. V. Lebedeva
Interaction of Negative Ions with the Surface of Inert Liquids, JETP Letters, 89(3),
145 (2009).**

**E. V. Lebedeva, A.M. Dyugaev, and P. D. Grigoriev, Diffusion Transport of
Negative Ions through the Interface between Cryogenic Liquids JETP, 110(4), 693
(2010).**

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Density-functional approach for investigation of electron bubble properties near the surface of liquid helium.

F. Ancilotto and F. Toigo, *Phys. Rev. B*, 50, 12820 (1994).



Complicated procedure of simulation.

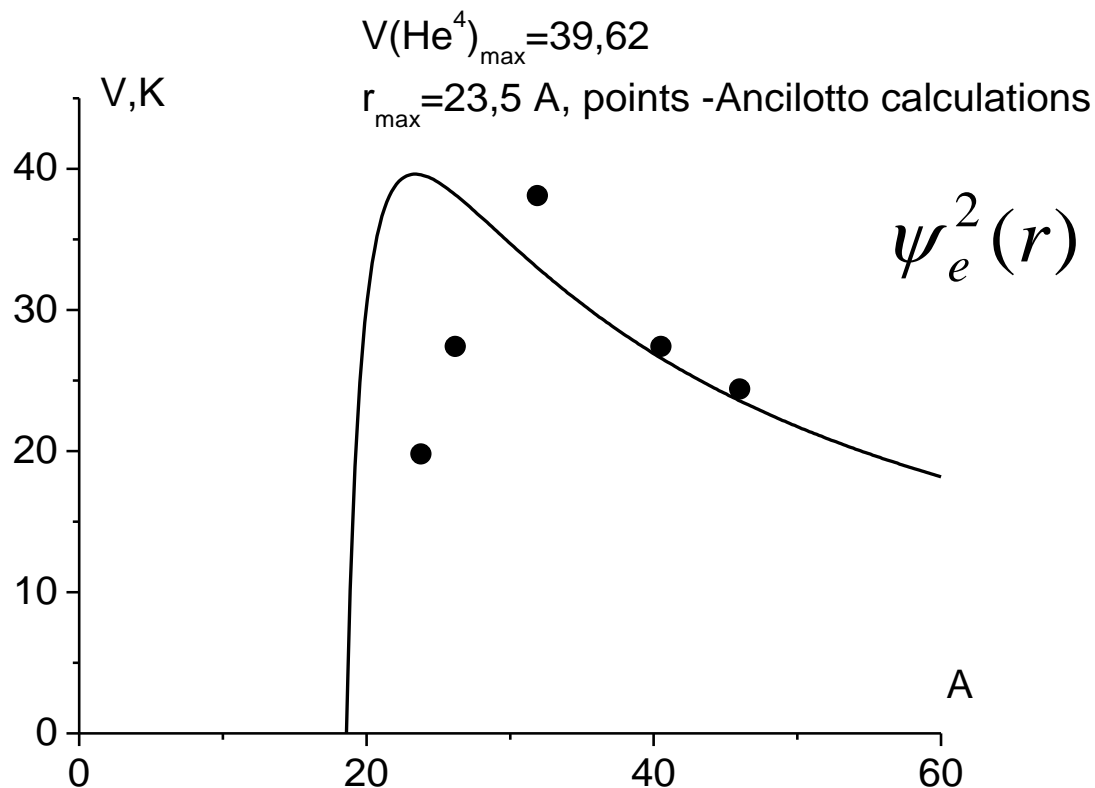
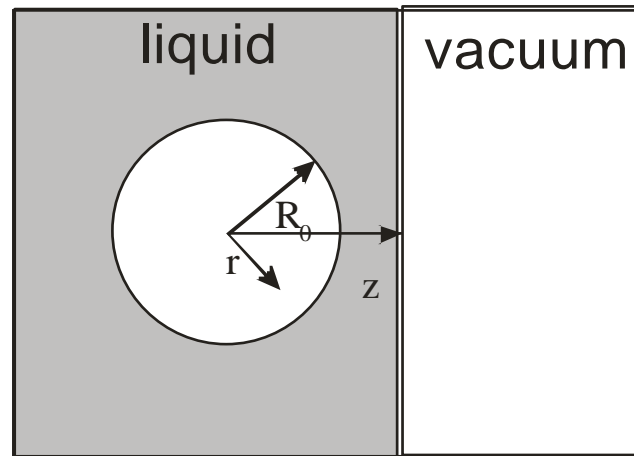
- definition of energy functional for electron and He;
- solution of nonlinear Schrödinger's equations; for electron and helium density functions
- energy minimization, laws of conservation;
- adjustment of procedure parameters to reproduce the experimental values of He constants.

Energy of helium atom: kinetic, He-He interaction, He-electron interaction.

Electron energy: kinetic, He-electron interaction.

**Our approach is continuous media approximation.
Instead of complicated computation we used the
simple integration on the half-space.**

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Inert Liquids, JETP Letters, 89(3), 145 (2009).**



$$\psi_e^2(r) = \frac{1}{4\pi R_0} \frac{2 \sin^2(\pi r / R_0)}{r^2}$$

$$V(\vec{r}_1 - \vec{r}_2) = -\frac{C_6}{|\vec{r}_1 - \vec{r}_2|^6}$$

$$V_e(z) = A \int \frac{\psi_e^2(r)}{z(r)}$$

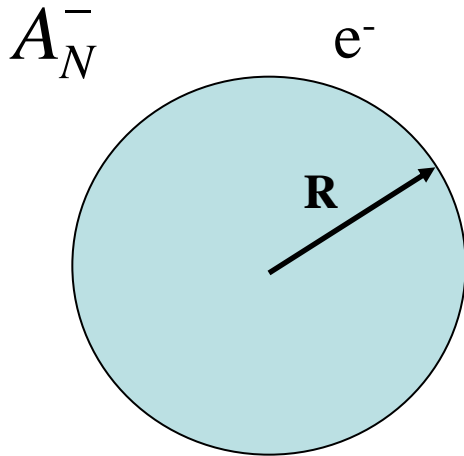
Charged cluster in vacuum (H_2 , Ne, He)

V.M. Nabutovskii and D.A. Romanov, *Sov. Phys. Low Temp. Phys.* 11, 277 (1987)

M Rosenblit and J Jortner *J. Chem. Phys.* 101, 9982, (1994).

M Rosenblit and J Jortner *J. Chem. Phys.* 101, 8039, (1994).

- Weakness of polarization attraction between electron and inert atom.
- The number N of atoms or molecules in the cluster, that could exist, is macroscopically large.
- Charged cluster can't exist if $R < R_c$.



$$\text{H}_2 : R_c = 35A, N_c = 4.6 \times 10^3$$

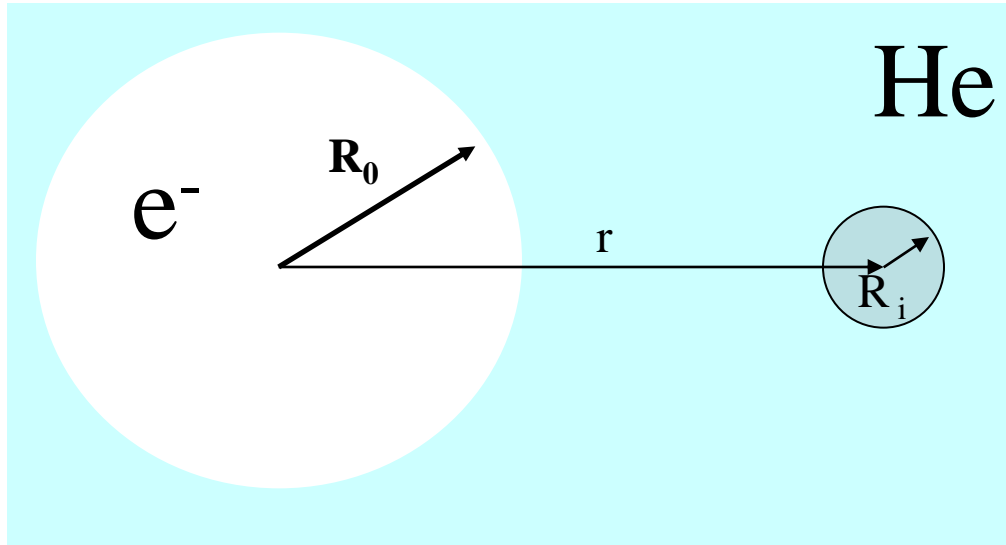
$$\text{Ne} : R_c = 39A, N_c = 1.5 \times 10^4$$

$$\text{He}^4 : R_c = 149A, N_c = 3 \times 10^5$$

$$\text{He}^3 : R_c = 203A, N_c = 5.7 \times 10^5$$

Charged Clusters in Liquid Helium.

Cluster-electron interaction potential: polarization and van der Waals parts



N_i – the number of particles in cluster,

$$10 < N_i < 1000$$

$N_0 = 450$ – the number of helium atoms, replaced by the bubble.

$$M_e = 900 \text{ a.m.u.}$$

$$R_0 = 17\text{\AA}, \quad R_i < R_0$$

Effective van der Waals repulsion potential:

$$v_{0i}(r) = \frac{e^2 c_{0i} r_B^5}{r^6}$$

Attractive potential of cluster's particle polarization by a point charge:

$$v_e = -\frac{e^2 \alpha_i r_B^3}{2r^4}$$

α_i - polarizability of cluster particles,

c_{0i} - van der Waals constant for cluster's particle-He interaction,

r_B - Bohr's radius.

Reduced interaction potential of cluster and electron.

$$V(r) = E_i V_-^*(x), \quad x = r/R_0, \quad E_i = \frac{e^2 N_i \alpha_i r_B^3}{2R_0^4}$$

$$V_-^*(x) = f_e(x) + \lambda f_b(x)$$

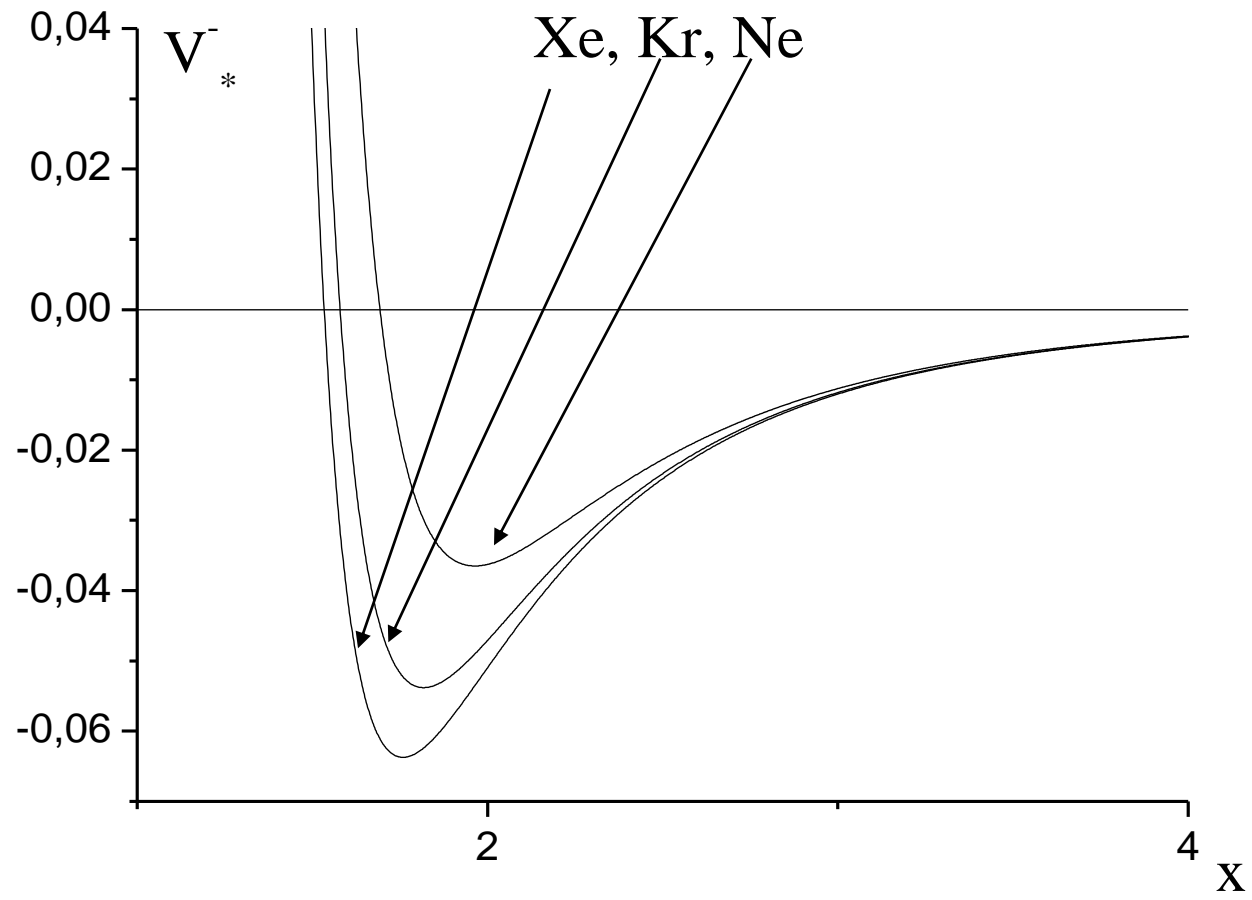
$$V_-^* = -\int_0^1 \frac{2 \sin^2 \pi y dy}{(x^2 - y^2)^2} + \frac{\lambda}{(x^2 - 1)^3}$$

$$\lambda = \frac{2N_0 r_B^2 c_{0i}}{R_0^2 \alpha_i} \approx 0.6 - 1.$$

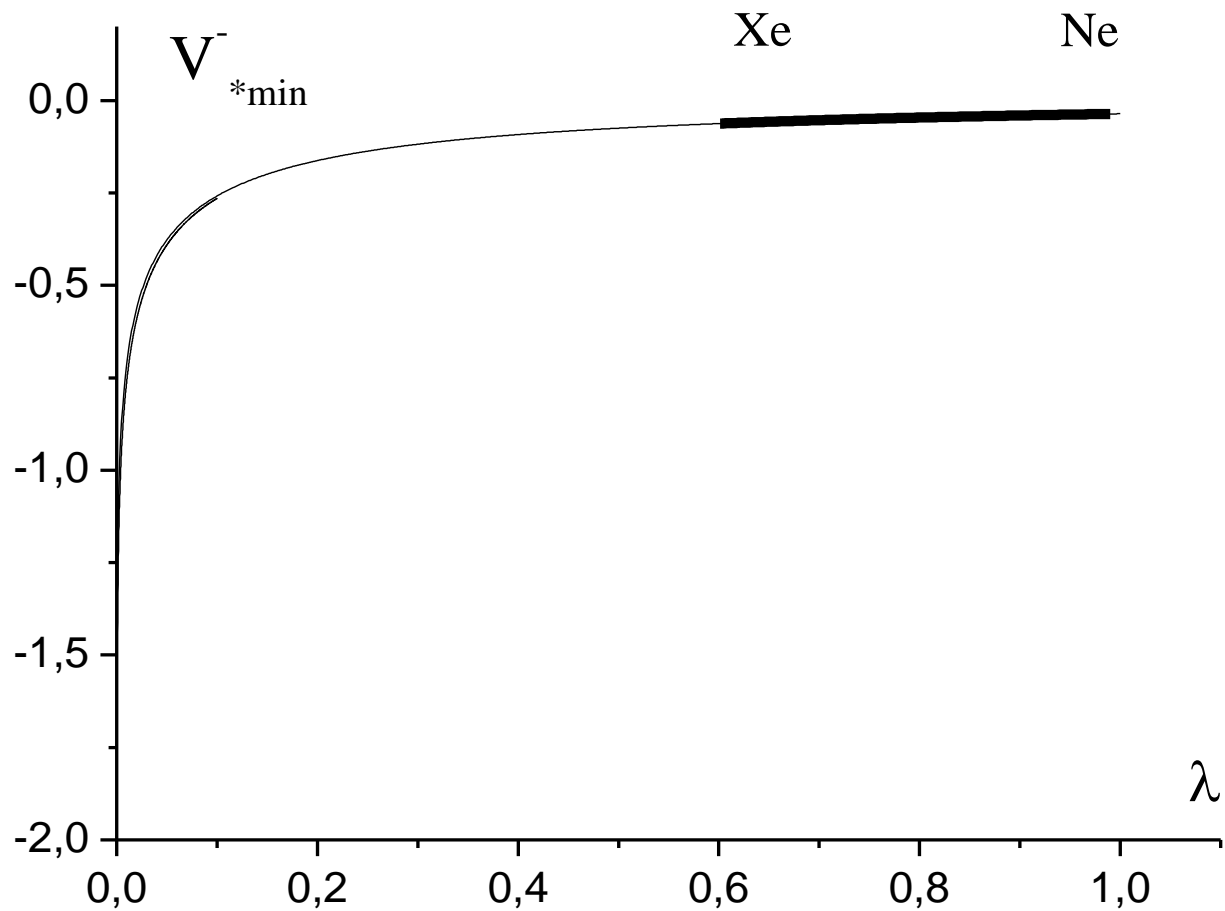
$$\frac{c_{0i}}{\alpha_i} \sim \frac{\Delta_i}{\Delta_i + \Delta_0} : 0.33 - 0.47$$

$$c_{0i} : 3 - 10 \quad \alpha_i : 3 - 27$$

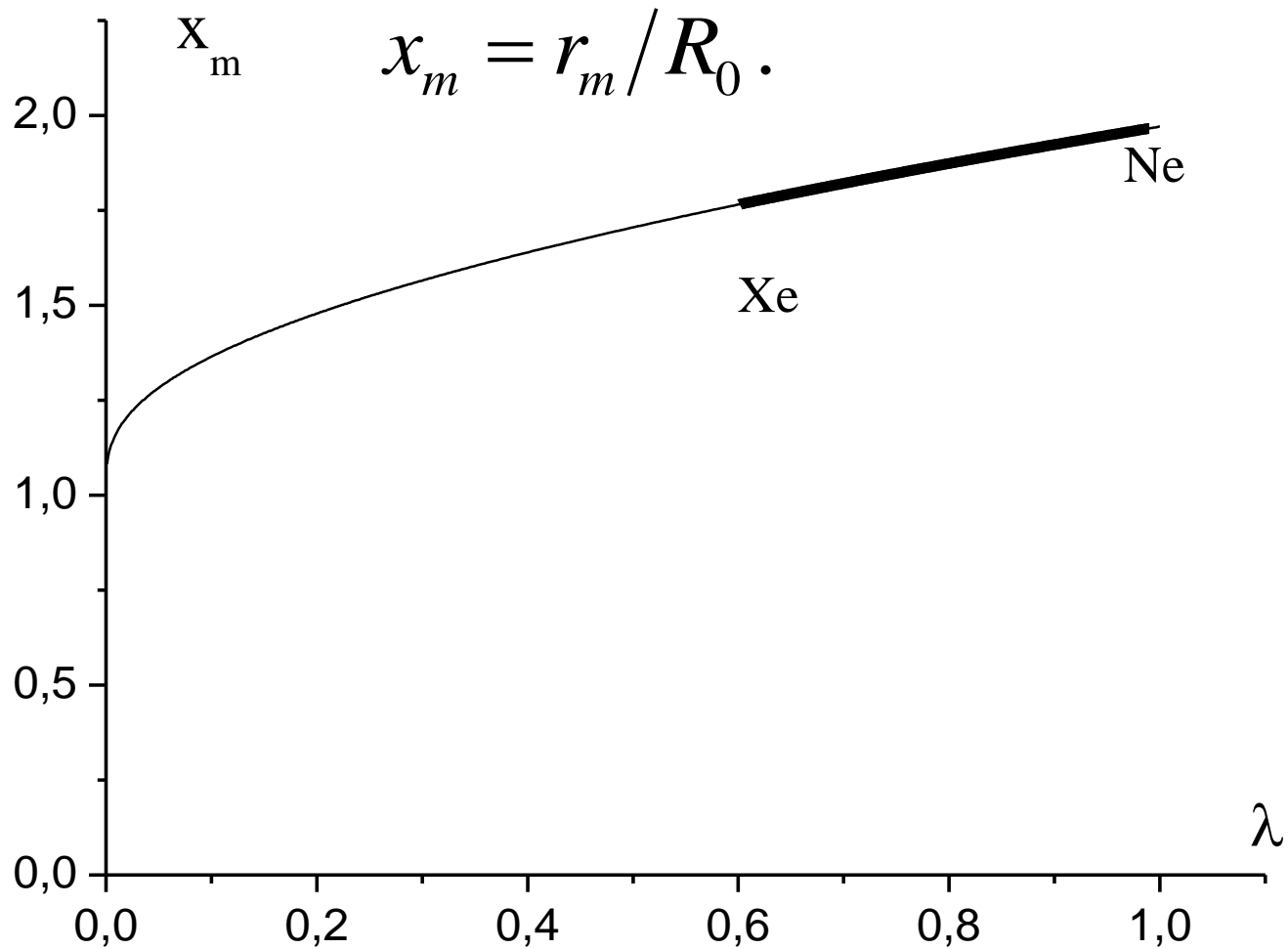
H. Margenau,
Rev. Mod. Phys. 11, 1-35 (1939).



The dependence of reduced potential minimum on λ -parameter



The dependence of reduced potential minimum position on λ -parameter



Charged cluster's characteristics in liquid helium

	α	c_{0i}	λ	V_{*min}^-	r_{min}	N_1	f_l	f_s
Ne	2,66	3,04	0,99	-0,036	33,4	70	0,119	0,036
Ar	11,1	9,60	0,752	-0,049	31,4	12	0,140	0,046
Kr	16,7	13,4	0,698	-0,054	30,9	7	0,146	0,049
Xe	27,3	18,6	0,593	-0,063	29,9	4	0,159	0,056
H ₂	5,33	4,22	0,689	-0,054	30,9	23	0,147	0,049
N ₂	11,9	10,14	0,741	-0,050	31,3	11	0,142	0,047

Number of states N_s and number of levels N_l

$$N_l = \frac{1}{8} N_i \frac{M_i^*}{m_e} \alpha_i \frac{r_B^2}{R_0^2} f_l, \quad f_l = \int \left(-V_*^-(x) \right) x dx$$

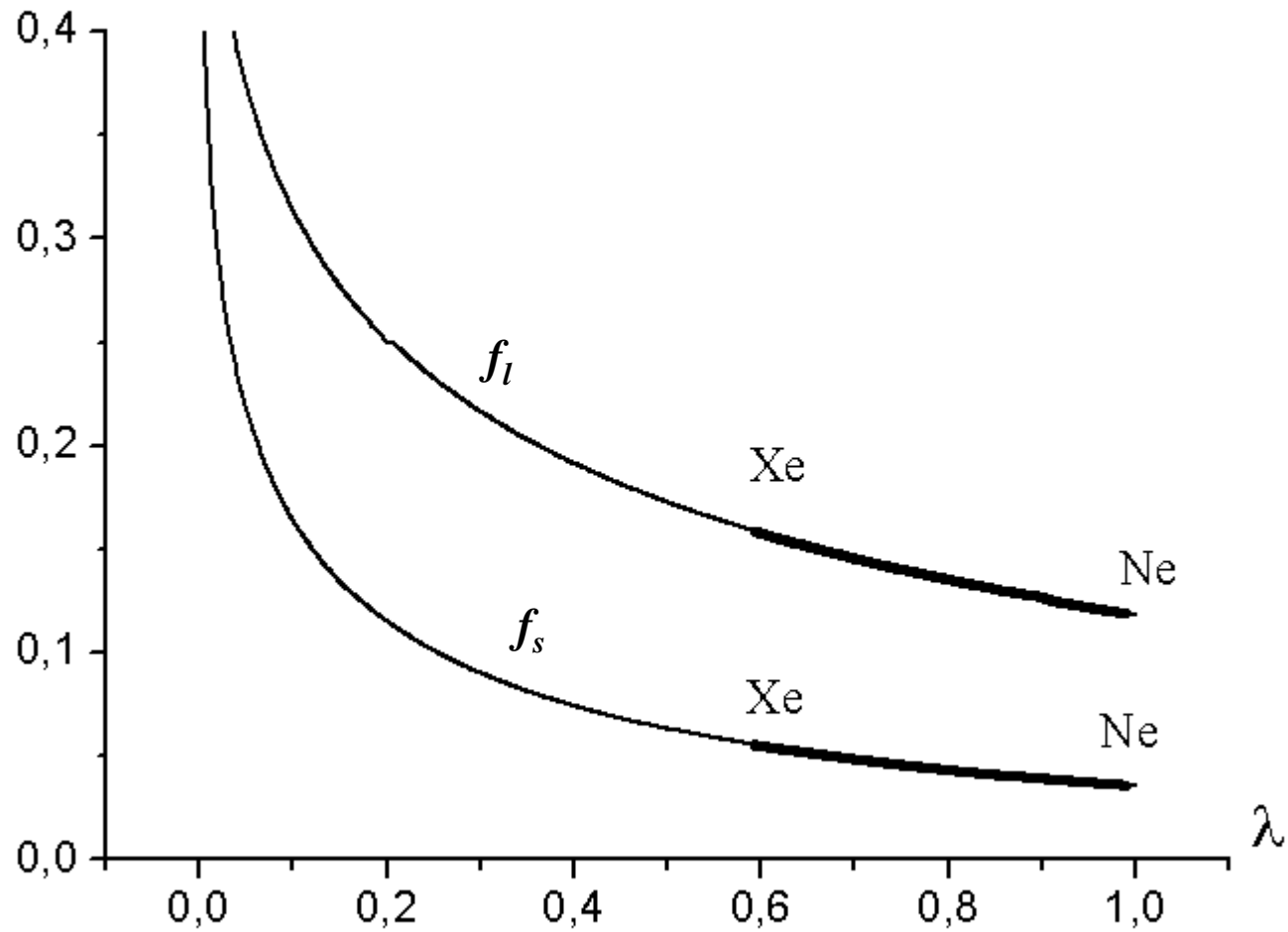
$$N_s = \frac{2}{3\pi} \left(N_i \frac{M_i^*}{m_e} \alpha_i \frac{r_B^2}{R_0^2} \right)^{3/2} f_s, \quad f_s = \int \left(-V_*^-(x) \right)^{3/2} x^2 dx$$

$$M_i^* = \frac{M_e M_i}{M_e + M_i}, \quad \frac{M_e}{m_e} = 1.64 \cdot 10^6, \quad M_e = 900 \text{ amu}$$

Example.

Xe: $N_i=13$, $M_i=1700$ amu. , $N_s=2.67 \times 10^6$, $N_l=7350$.

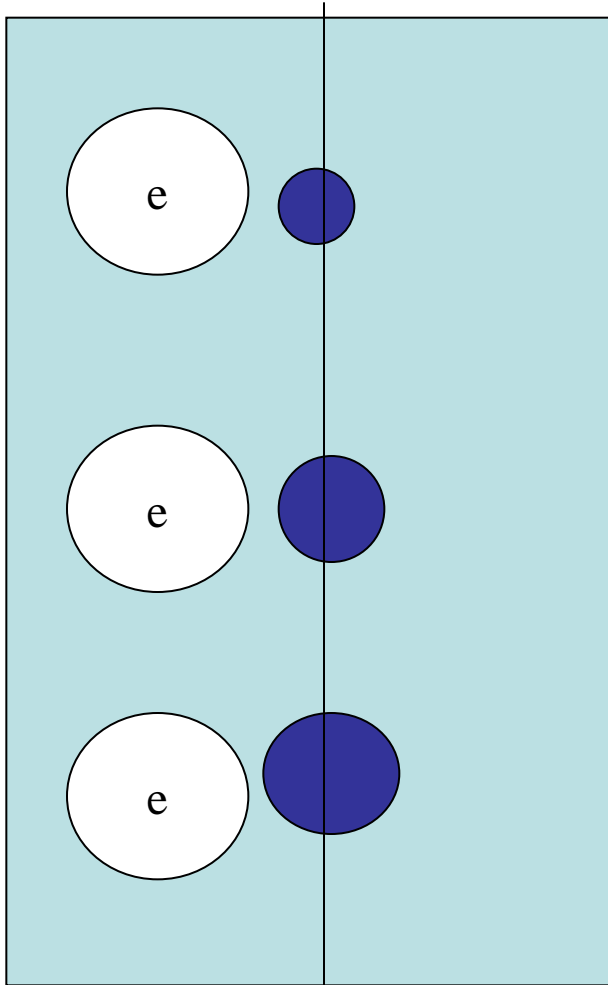
The dependence of reduced potential's parameters f_l f_s on λ



Charged cluster's characteristics in liquid helium

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$$R_i = 13 - 16 \text{ \AA}$$



Ground state energy of charged cluster:

$$\varepsilon_{i0} = E_i V_{-\min}^*$$

$$\text{Ne: } R_i = 16 \text{ \AA}$$

$$N_i \approx 1000$$

$$\varepsilon_{i0} \approx 15 \text{ K}$$

$$H_2 : R_i = 14 \text{ \AA}$$

$$N_i \approx 300$$

$$\varepsilon_{i0} \approx 13 \text{ K}$$

Internal energy of electron:

$$\varepsilon_e = \frac{\pi^2 \hbar^2}{m_e R_0^2} \approx 2 \cdot 10^3 \text{ K}$$

Summary

- **The existence of small charged inert clusters formed by Ne, Ar, Kr and Xe atoms and H₂ and N₂ molecules had been proved.**
- **The properties of these clusters had been semi-classically described.**
- **The scaling behavior of cluster parameters had been found out.**

