Electron-phonon interaction from Migdal to our days

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ВЗАИМОДЕЙСТВИЕ ЭЛЕКТРОНОВ С КОЛЕБАНИЯМИ РЕШЕТКИ В НОРМАЛЬНОМ МЕТАЛЛЕ

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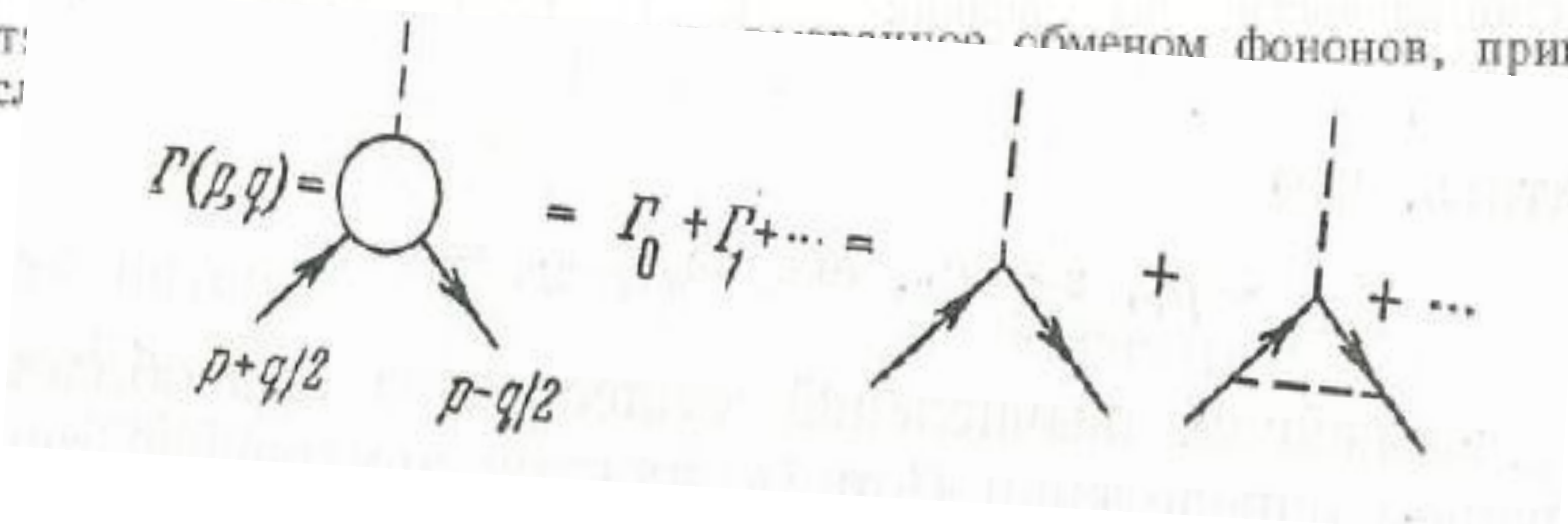
Развит метод расчета, позволяющий найти энергетический спектр электронов и дис-
ию колебаний решетки без предположения о малости взаимодействия между электро-
и фононами.

1. Введение

Прит … обменом фононов, приво-
в с … ух



Seminal paper

Migdal [1] developed a consistent many-body approach based on the Fröhlich Hamiltonian for interaction of electrons with acoustic (sound) phonons. As Migdal showed ("the Migdal theorem"), the vertex corrections for acoustic phonons are small by the *adiabatic* parameter $\sqrt{m/M}$, where m and M are the electron and ion masses, respectively. The theory described correctly the electronic lifetime, renormalization of the Fermi velocity v_F and acoustic phonon attenuation but resulted in a strong renormalization of the sound velocity $\tilde{s} = s(1 - 2\lambda)^{1/2}$, where λ is the dimensionless coupling constant. For sufficiently strong electron-phonon coupling $\lambda \to 1/2$, the phonon frequency approached to zero marking an instability point of the system. Instead, one would intuitively expect the phonon renormalization to be weak along with the adiabatic parameter.

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Adiabatic parameter

$$M\omega^2 u^2 \sim \frac{e^2}{a^3} u^2$$

$$\varepsilon \sim \frac{e^2}{a} \sim \frac{p^2}{m} \qquad e^2 \sim \frac{1}{am}$$

$$\frac{\omega}{\varepsilon} \sim \sqrt{\frac{e^2}{Ma^3} a^4 m^2} \sim \sqrt{\frac{m}{M}}$$

electron–acoustic-phonon interaction

$$\varepsilon(\mathbf{p},\mathbf{r},t) = \varepsilon_0(\mathbf{p}) + \zeta_{ik}(\mathbf{p})u_{ik}(\mathbf{r},t)$$

u_{ik} is the strain tensor,

ζ_{ik} is the deformation potential.

sound-velocity shift and attenuation

$$\frac{\delta s}{s} - i\frac{\Gamma}{\omega_k} = \lambda \begin{cases} \left(\dfrac{s}{v_F}\right)^2 - i\dfrac{\pi s}{2v_F}, & kv_F > |\omega_k + i\gamma| \\ \dfrac{\pi\omega_k}{\omega_k + i\gamma}, & kv_F < |\omega_k + i\gamma| \end{cases}$$

γ is the electronic scattering rate,

$\lambda = \zeta^2\nu_0/\pi\rho s^2$ is the dimensionless coupling

ν_0 is the electronic density of states

ρ is the metal density

Phonon - plasmon modes

$$\varepsilon(\mathbf{p},\mathbf{r},t) = \varepsilon_0(\mathbf{p}) + \zeta(\mathbf{p})\nabla\cdot\mathbf{u}(\mathbf{r},t)$$

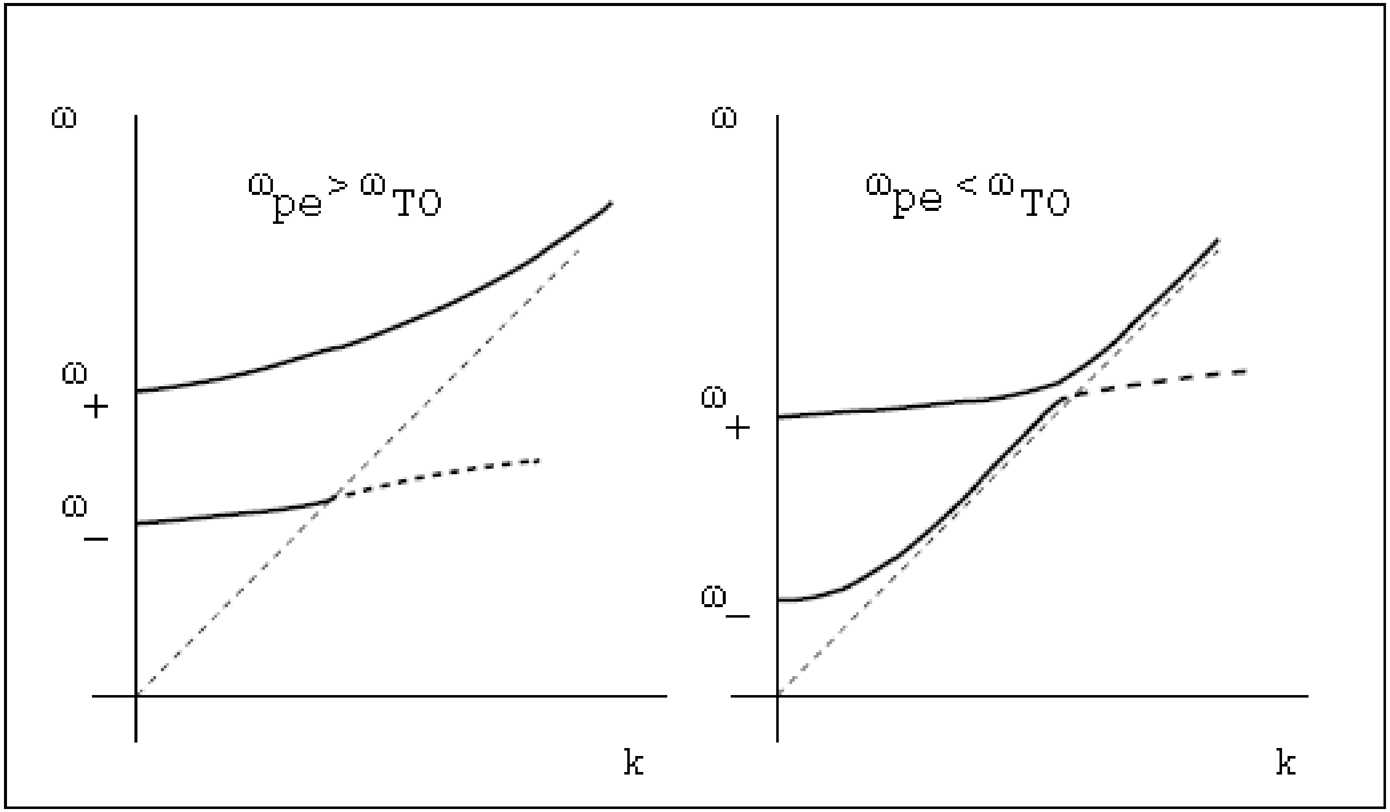
$$(\omega_k^2 - \omega^2)u_j(\mathbf{k},\omega) = \frac{Z}{M'}E_j + \frac{ik_j}{M'N}\int\frac{2d^3p}{(2\pi)^3}\zeta(\mathbf{p})\delta f_p(\mathbf{k},\omega)$$

$$-i(\omega - \mathbf{k}\cdot\mathbf{v})\delta f_p(\mathbf{k},\omega) + \gamma\left[\delta f_p(\mathbf{k},\omega) - \langle\delta f_p(\mathbf{k},\omega)\rangle\right]$$
$$= -[\omega\zeta(\mathbf{p})\mathbf{k}\cdot\mathbf{u}(\mathbf{k},\omega) + e\mathbf{v}\cdot\mathbf{E}]\frac{df_0}{d\varepsilon}$$

$$\mathbf{E} = -4\pi\mathbf{k}(\mathbf{k}\cdot\mathbf{P})/k^2$$

$$P_z = NZu_z + \alpha E + \frac{ie}{k}\int\frac{2d^3p}{(2\pi)^3}\delta f_p(\mathbf{k},\omega).$$

Phonon - plasmon modes



Renormalization of the Fermi velocity due to el-el interactions