ELUCIDATION of NFL BEHAVIOR within a LANDAU APPROACH

Two Basic Postulates:

I. The energy E, entropy S and other thermodynamic quantities are functionals of the quasiparticle momentum distribution $n(p)$, the number of quasiparticles being equal to the particle number:

$$\mathrm{Tr} \int n(\mathbf{p})\, dv = \frac{N}{V} \equiv \rho$$

II. The entropy S is given by a combinatorial expression

$$S = -\mathrm{Tr} \int [n(\mathbf{p}) \ln n(\mathbf{p}) + (1 - n(\mathbf{p})) \ln(1 - n(\mathbf{p}))] dv$$

- A standard variational procedure with additional conditions $\delta N = 0$ and $\delta E = 0$ leads to the Fermi-like distribution function

$$n(p) = \left(1 + e^{\epsilon(p)/T}\right)^{-1} \rightarrow \epsilon(p) = \delta\Omega/\delta n(p) \rightarrow \Omega = E - \mu N$$

- Landau (1956)

  Quasipartticle energy ϵ being a functional of n depends on temperature T as well

  In accordance with the formula for n, the specific heat $C(T)$ is proportional to T

- Additional Assumption:

  Fermi surface is simply connected ("Adiabaticity" of switching of interactions)

  Indeed, any Fermi liquid with the simply connected Fermi surface behaves as a GAS of interacting quasiparticles:

$$n(p, T = 0) = \theta(p_F - p)\ , \quad \epsilon_{FL}(p) = v_F(p - p_F)\ , \quad v_F = \frac{p_F}{M^*} > 0$$

$$C(T \rightarrow 0) \propto M^* T, \quad \chi(T \rightarrow 0) = const \propto M^*$$

Verification of this Assumption in Quantitative FL theory

- The quasiparticle spectrum $\epsilon(p)$ is evaluated from the Landau relation

$$\frac{\partial \epsilon(p)}{\partial \mathbf{p}} = \frac{\mathbf{p}}{M} + \int f(\mathbf{p}, \mathbf{p_1}) \frac{\partial n(p_1)}{\partial \mathbf{p_1}} \, dv_1,$$

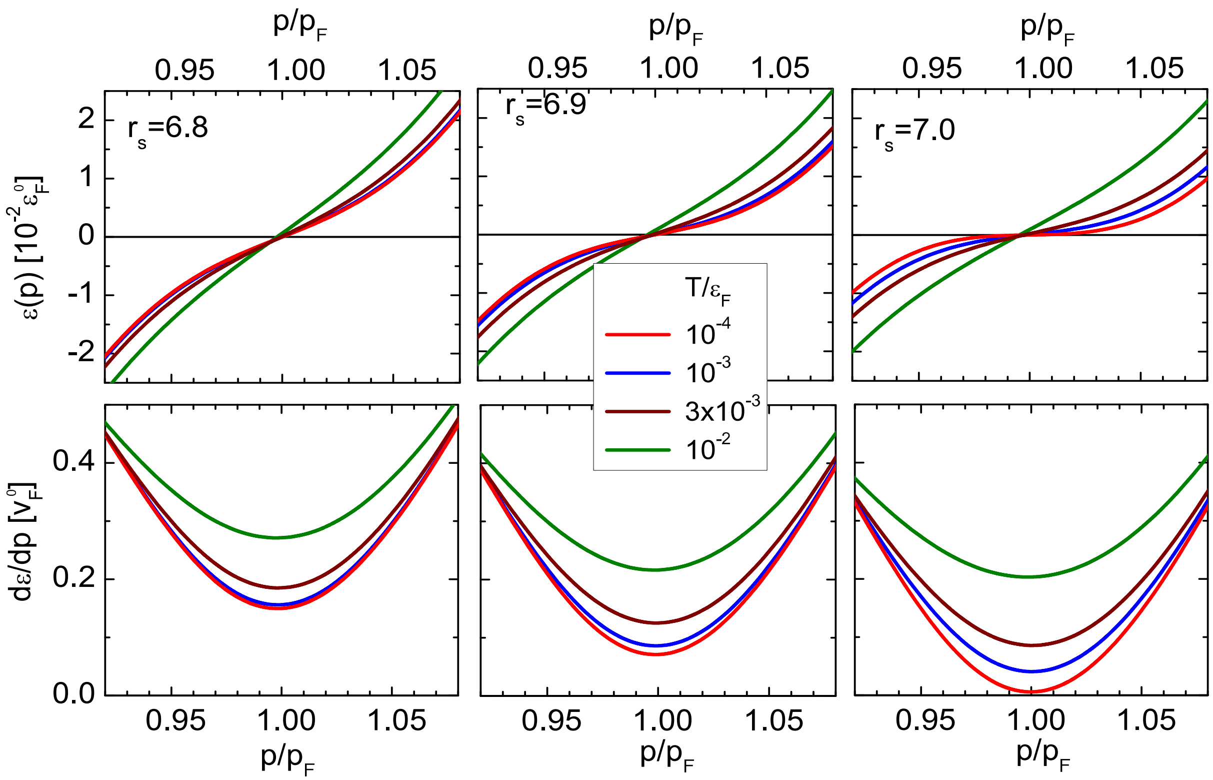
treating the interaction function $f(\mathbf{p}, \mathbf{p}_1)$ as phenomenological input.

$$\frac{M}{M^*} = \frac{v_F(\rho)}{v_F^0} = 1 - \frac{1}{3} F_1^0(\rho)$$

where $v_F^0 = p_F/M$ and $F_1^0 = f_1(\rho) p_F M/\pi^2$.

- Critical condition for connectivity of the Fermi surface

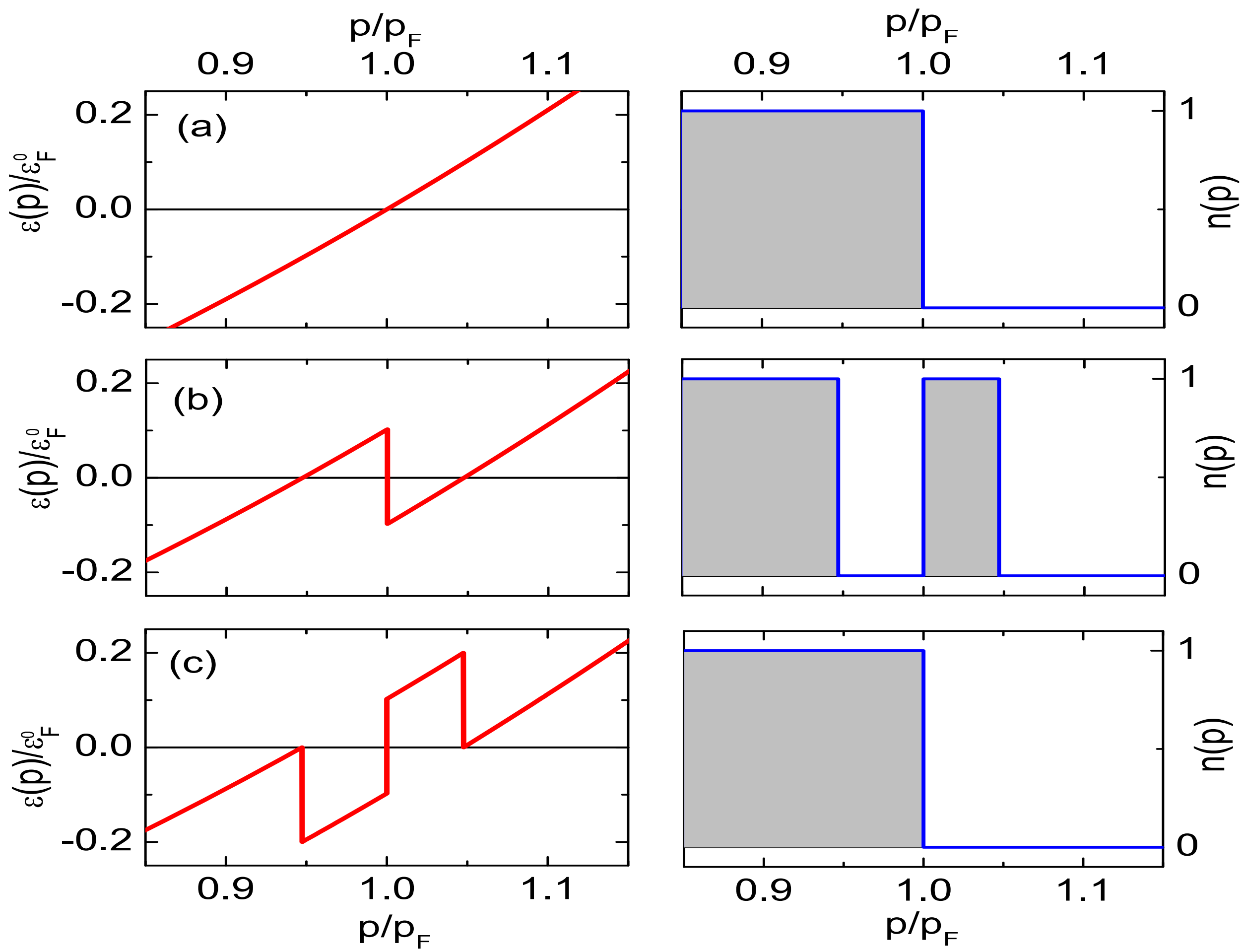
$$v_F(\rho_\infty) = 0 \quad \rightarrow M^*(\rho_\infty) = \infty$$



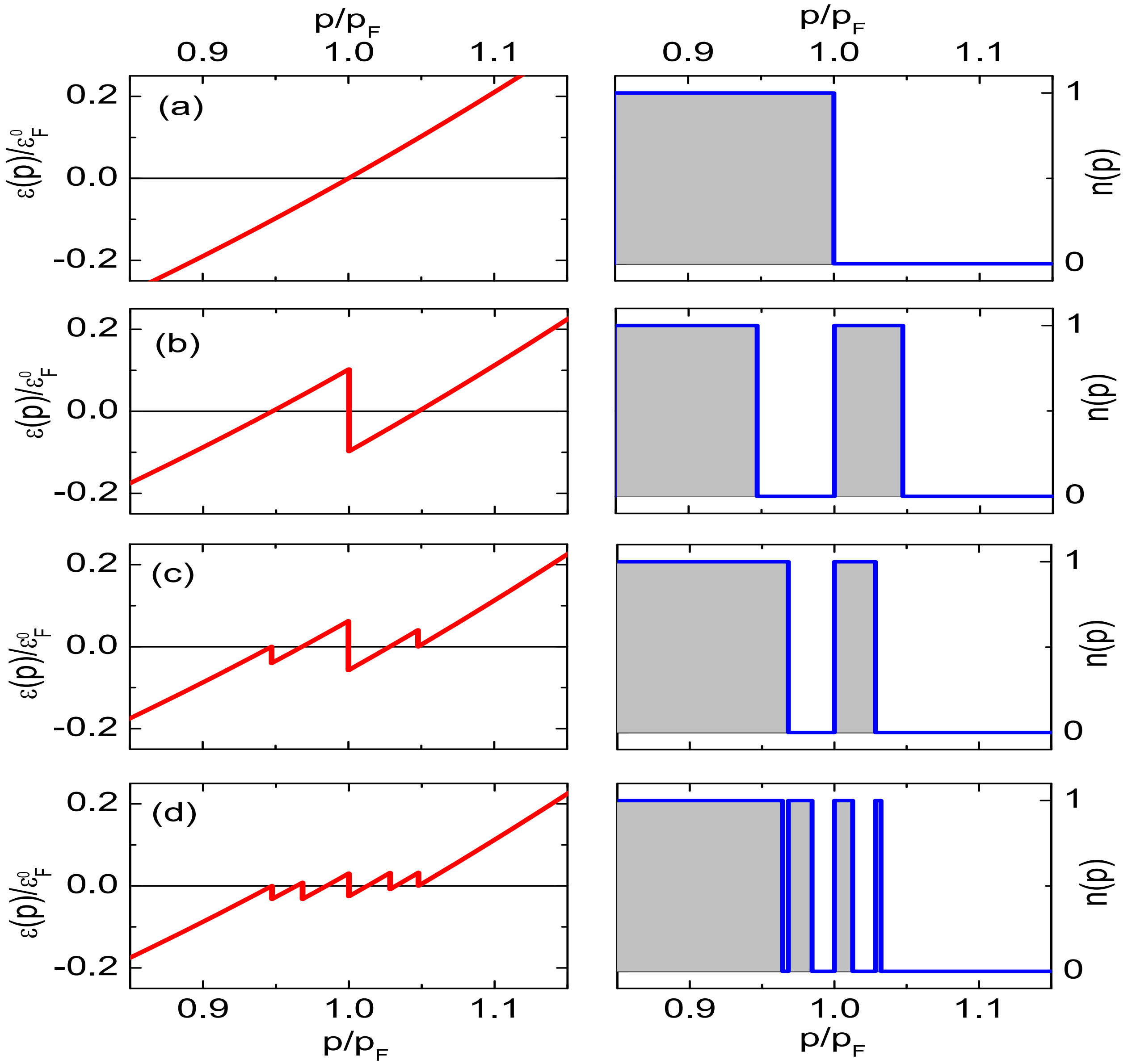
$f(k) = -g(\pi/M)\left[(k^2/4p_F^2 - 1)^2 + \beta^2\right]^{-1}$, with $g = 0.16$, $\beta = 0.14$, and $r_s \propto r_0/a_B$.

the spectrum $\epsilon(p)$ does depend on T

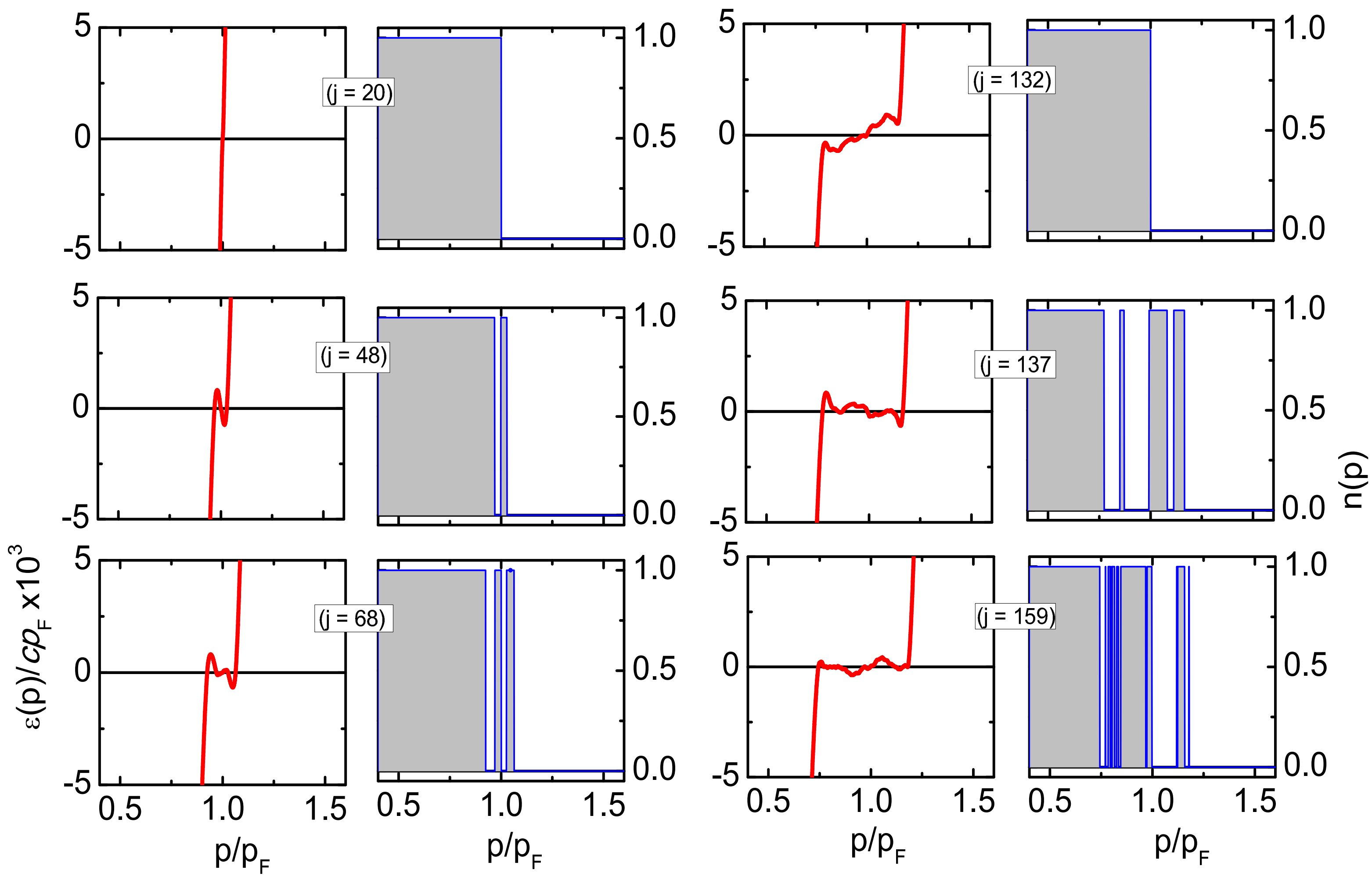
Nonconvergent Iteration Procedure



- Nozieres model $\epsilon(p) = p^2/2M + fn(p)$



p/p$_F$
0.9
1.0
1.1
(a)
(b)
(c)
(d)
0.2
0.0
-0.2
ε(p)/ε^0_F
n(p)
1
0



- Nonzero Entropy !!

New Ground State beyond the Critical Point

- Landau ground state with $n_F(p) = \theta(p_F - p)$ holds as long as

  the necessary stability condition

$$\delta E_0 = \int \epsilon(p; n_F(p)) \delta n(p) dv > 0,$$

is not violated. Beyond the critical point, the single-particle spectrum $\epsilon(p)$

  and variations $\delta n(p)$ have opposite signs, and therefore $\delta E_0 < 0$.

- Khodel,Shaginyan (1990):

  Equation for the quasiparticle momentum distribution:

$$\frac{\delta E(n)}{\delta n(p)} - \mu = 0 \quad \rightarrow \epsilon(p) = 0, \quad p \in \mathcal{C}$$

- Volovik (1991):

  Topological charge

$$\mathcal{N} = \int\limits_{\gamma} G(p,\varepsilon)\,\partial_l G(p,\varepsilon)\frac{dl}{2\pi i}\ ,$$

  Relevant Green function has the form:

$$G(p,\epsilon) = \frac{1-n_*(p)}{\epsilon + i\delta} + \frac{n_*(p)}{\epsilon - i\delta} \qquad p \in \mathcal{C}$$

$\mathcal{N} = 1$ for conventional Fermi liquids, with simply-connected Fermi surface

  while for the states with fermion condensate $\mathcal{N} = 1/2$.

- Nozieres (1992)

  Finite temperatures $\longrightarrow$ (cf. Landau (1956))

$$\epsilon(p,T) = T \ln \frac{1-n_*(p)}{n_*(p)} \quad p \in \mathcal{C}$$

. Why "Fermion Condensation" ?

- Topological transition, where the Fermi surface swells from a line to a surface in 2D

  and from a surface to a volume in 3D is called fermion condensation.
- Merging of single-particle levels-analog of fermion condensation in finite systems.
- In everyday life, the term condensation means simply dramatic increase of density.
- In statistical physics, Bose condensation is the emergence of a macroscopic number of Bose particles, possessing the same single-particle energy $\epsilon = 0$.

  In strongly correlated Fermi systems, such a condensation is possible as well.

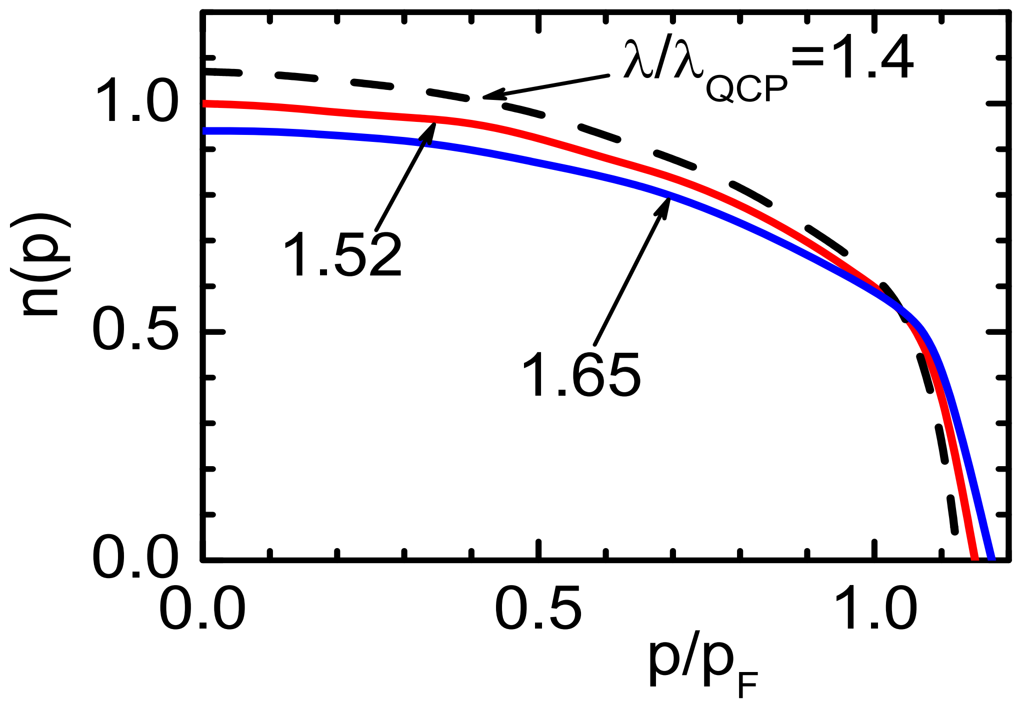
  A macroscopic number of fermions can have the same energy $\epsilon = 0$.
- Bose condensation exhibits itself in a sharp peak in the density of states:

$$\rho(\epsilon) = \rho_c \delta(\epsilon)$$

In systems with a FC, the $T = 0$ density of states has the same peak.

Solution of $T = 0$ Landau Equation beyond the Critical Point

- Solutions, consistent with the Pauli principle $n(p) < 1$, exist at $\lambda > \lambda_{cr}$.



$f(q) = \lambda(q^2 + \kappa^2)^{-1}$ with $\kappa = 0.07\, p_F$.

Thermodynamics of Systems with a Fermion Condensate

- Standard FL picture is violated .

I. Entropy

$$S(T \to 0) = S_* = -\int [n_*(p) \ln n_*(p) + (1 - n_*(p) \ln(1 - n_*(p))] dv > 0$$

has a finite value instead of $S_{FL}(0) = 0$.

The presence of the excessive entropy S_* shows itself in a huge enhancement

of the thermal expansion $\beta(T) \propto \partial S(T)/\partial \rho = const$ instead of $\beta_{FL}(T) \propto T$.

II. Spin susceptibility

$$\chi(T) = \frac{1}{T} \int n_*(p)(1 - n_*(p)) dv = \frac{C_{eff}}{T} \propto \frac{\rho_{FC}}{T}$$

exhibits Curie-like behavior instead of $\chi_{FL}(T \to 0) = const$.

III. Sommerfeld-Wilson ratio

$$R_{SW}(T) = \frac{\chi(T)}{\gamma(T)} \propto \frac{\rho_{FC}}{T}$$

diverges at $T \to 0$ instead of $R_{SW}(T) = const.$

IV. In the phase with the fermion condensate, particle-hole symmetry breaks down.

V. In systems with a fermion condensate, P-parity is violated.

Pairing in Systems with a Fermion Condensate

- Pairing lifts degeneracy, associated with the fermion condensate,

  and eliminates contradiction with the Nernst theorem.

  The BCS gap equation

$$\Delta(p,T)=\int \mathcal{V}(p,p_1)\frac{\tanh\frac{E(p_1)}{2T}}{2E(p_1)}\Delta(p_1,T)dv_1\ .$$

- Setting $\mathcal{V}(p,p_1)=g$ one finds $T_c=0.57\Delta(0)$ where

$$\Delta(0)=\Omega_D e^{-\frac{2}{gN(0)}}\ ,$$

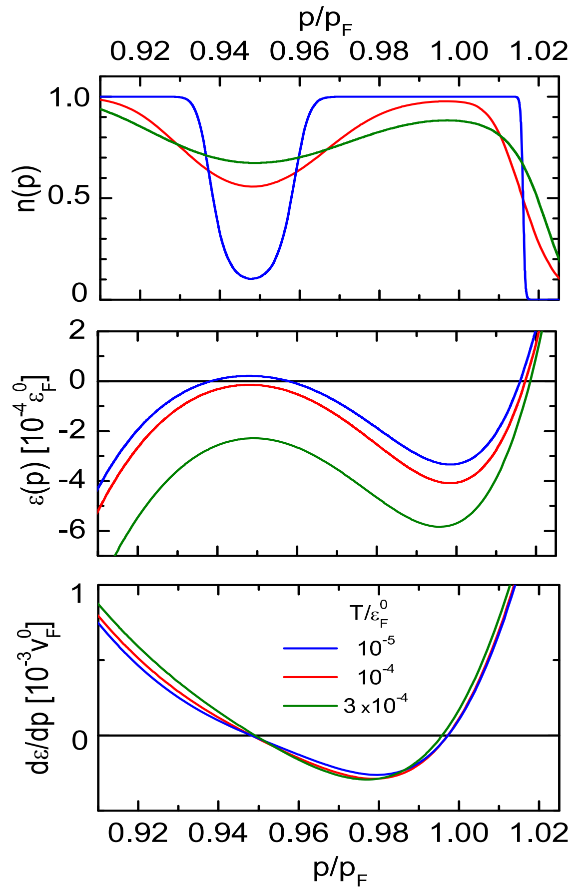
- In systems with fermion condensate $\Delta(0)\propto g$ (Khodel-Shaginyan 1990).

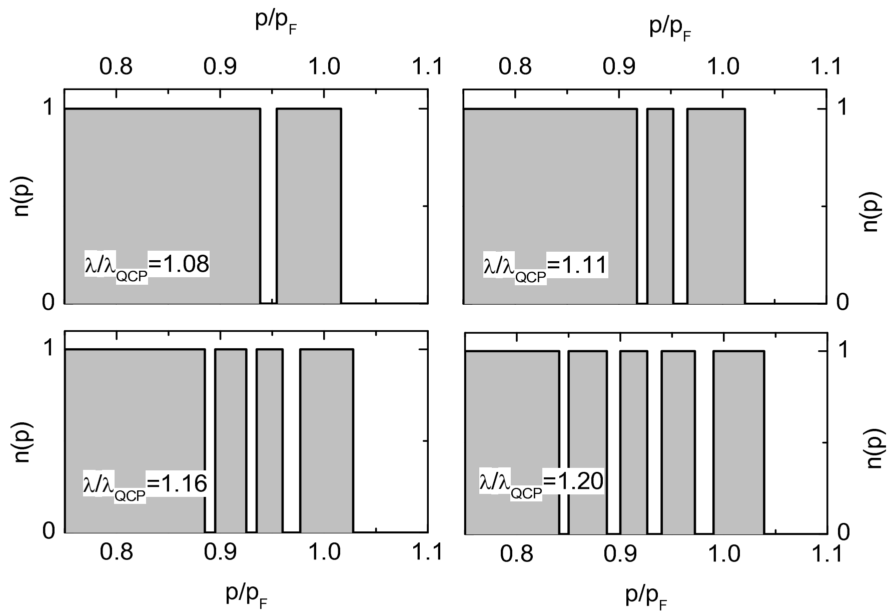
  Does this linearity open the way to high-T_c superconductivity?? (Pseudogap!)

Lifshitz Phace Diagram:

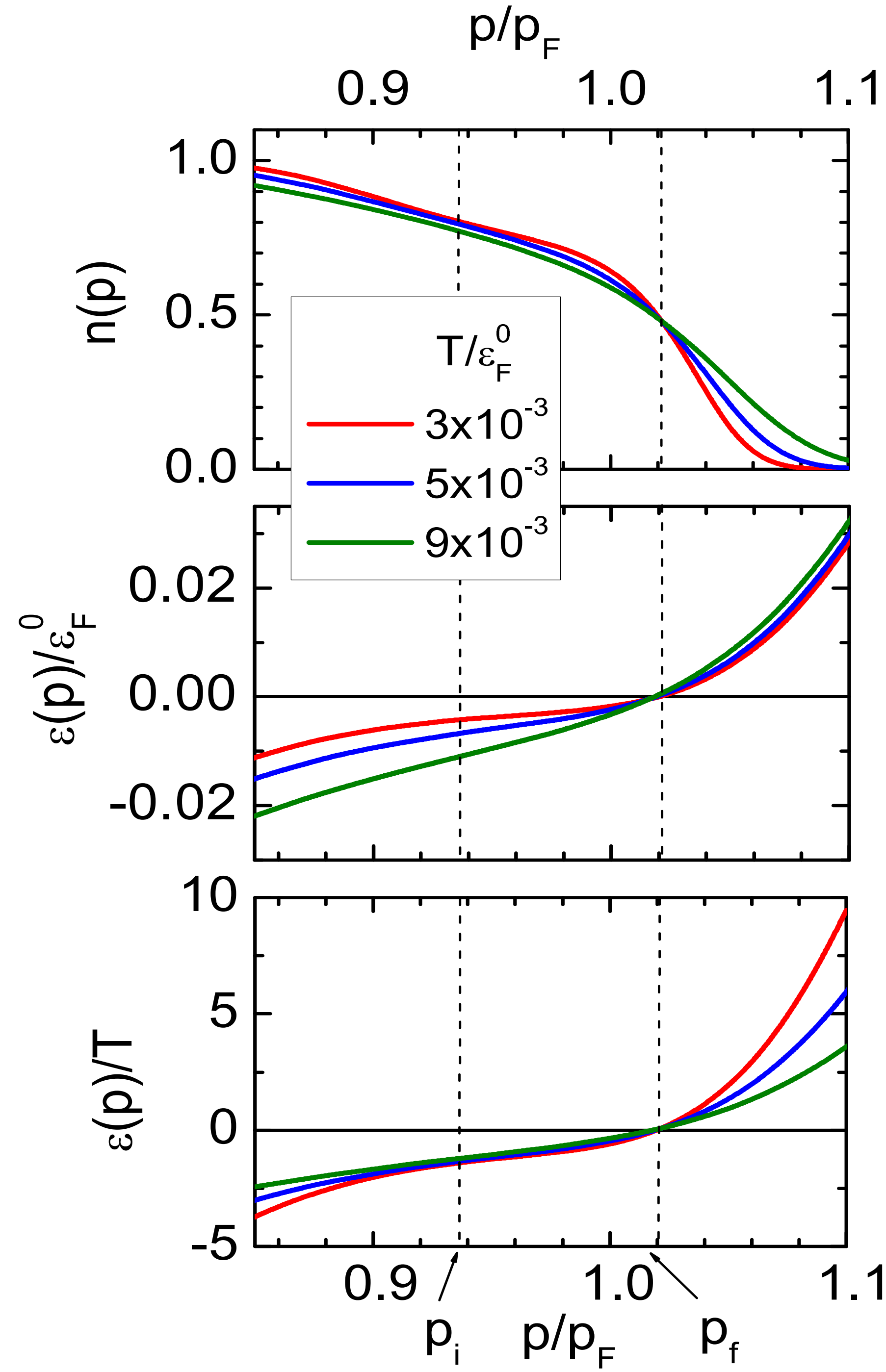
Between the Landau State state and a State with a Fermion Condensate

- Zverev,Baldo (1999)





p/p$_F$
0.8
0.9
1.0
1.1
n(p)
1
0
λ/λ$_{QCP}$=1.08
λ/λ$_{QCP}$=1.11
λ/λ$_{QCP}$=1.16
λ/λ$_{QCP}$=1.20



p/p$_F$
0.9
1.0
1.1
n(p)
1.0
0.5
0.0
T/ε$_F^0$
3x10^{-3}
5x10^{-3}
9x10^{-3}
ε(p)/ε$_F^0$
0.02
0.00
-0.02
ε(p)/T
10
5
0
-5
p$_i$
p/p$_F$
p$_f$

Occupation numbers $n(p)$, single-particle spectrum $\epsilon(p)$, and ratio $\epsilon(p)/T$.