Form Factors and Correlation functions

in perturbed Parafermionic CFT

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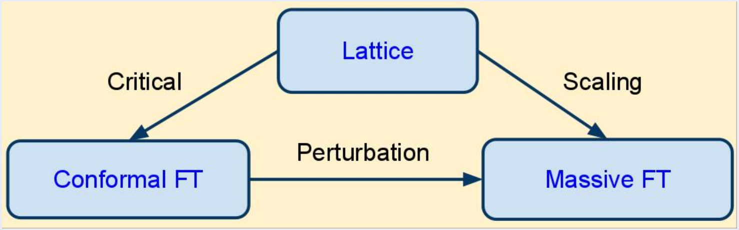
Integrable Models on Lattice and in scaling

Two Dimensional Integrable Models

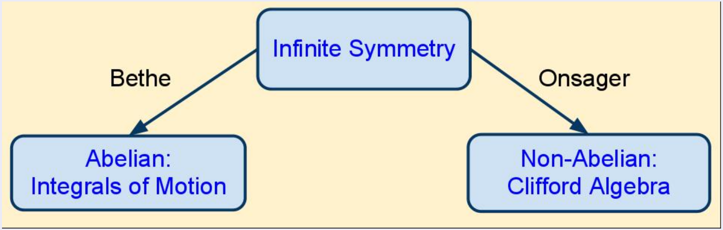


Sources

The talk is based on joint works with V.A. Fateev and V. Postnikov (2006-2010) and with M. Lashkevich (2011)

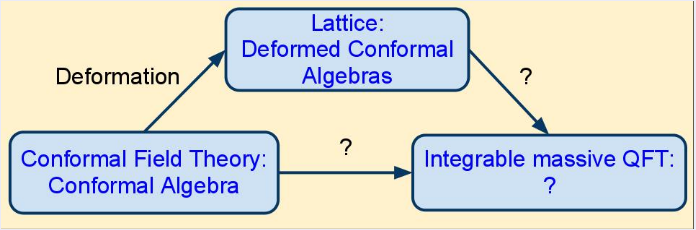
Two Dimensional Integrable Models

Integrability



Non-Abelian Symmetry

Infinite-dimensional algebras

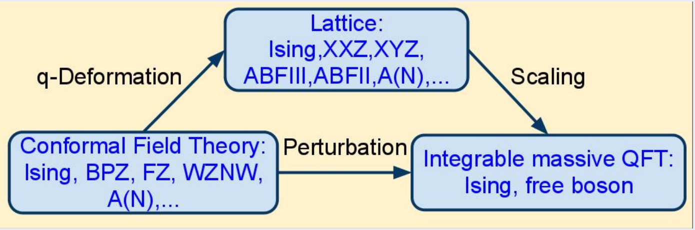


Space of states. Identical results for

- CFT - character of conformal algebra irreducible representation
- Off critical Lattice model - trace of corner transfer matrix
- Massive QFT - number of solutions of form factor equations

Exact correlation functions

Exact answers for correlators

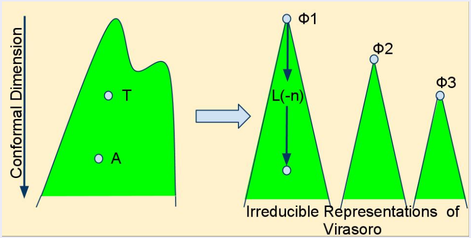


Space of States in CFT

Virasoro Algebra

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}(n^2-1)n\delta_{n+m}, \quad T(z) = \sum L_n z^{-n-2}$$

Primaries Φ and Descendants $A_j = L_{-n_1} \cdots L_{-n_a} \bar{L}_{-n_1} \cdots \bar{L}_{-m_b} \Phi$



Algebraic approach to CFT and lattice models

Dynamical symmetry algebras in CFT

- Chiral space of states is a direct sum of irreps of Virasoro algebra and its extensions: Affine, W, Parafermionic, Super-conformal
- Chiral fields intertwine these irreps. Their matrix elements are building blocks for correlators

Dynamical symmetry algebras in off-critical lattice models

- The algebraic approach in integrable off-critical lattice model in the Corner Transfer matrix approach is similar
- The algebras are the deformed conformal algebras.

Integrable massive Z(N) symmetric Ising model

Different descriptions

- Lattice model description
- Perturbed Parafermionic CFT ($c = 2\frac{N-1}{N+2},\ \Delta_\varepsilon = \frac{2}{(N+2)}$)

$$\mathcal{A} = \mathcal{A}_{CFT} + \lambda \int d^2x \ \varepsilon(x)$$

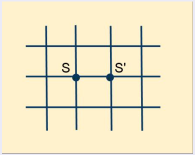
- S-matrix description

$$S(\beta) = \sinh(\frac{\beta}{2} + \frac{i\pi}{N})/\sinh(\frac{\beta}{2} - \frac{i\pi}{N})$$

Lattice Z(N) symmetric Ising model (N=2,3,4,. . .)

General model $s \in \{1, \omega, \ldots, \omega^{N-1}\}, \omega = e^{\frac{i\pi}{N}}$

$$Z = \sum e^{-H}$$



$$e^{-H(s,s')} = \sum_{0 \le k < N} w_k (s' s^{\dagger})^k$$

Z_N symmetry

$$H(s, s') = H(\omega s, \omega s')$$

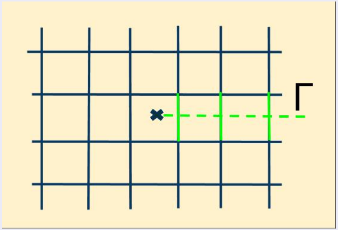
The Krammers-Wannier duality $\tilde{w}_k = \sum_s \omega^{ks} w_s / \sum_s w_s$

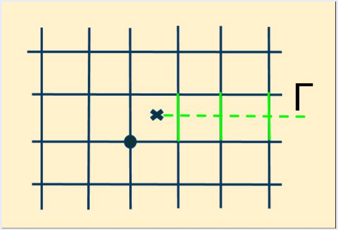
Order s_k operators

$$\langle s_k \rangle = \sum s^k e^{-\beta H} / Z$$

Disorder μ_k and composite semi-local operators

H_Γ : $w_s \longrightarrow \omega^{-ks} w_s$, along green lines

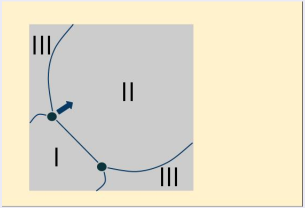




$$\langle \mu_k \rangle = \sum e^{-\beta H_\Gamma} / Z, \qquad \langle s_k \mu_k \cdots \rangle \longrightarrow \langle \psi_k \cdots \rangle$$

Integrable models

Phase Diagramm N=5 Model



- I $\langle s_j \rangle \neq 0, \quad \langle \mu_j \rangle = 0$
- II $\langle s_j \rangle = 0, \quad \langle \mu_j \rangle \neq 0$
- III $\langle s_j \rangle = 0, \quad \langle \mu_j \rangle = 0$

Parafermionic CFT

Dynamical symmetries of CFT with $c = 2(N-1)/(N+2)$

- Dimensions of σ_k and μ_k are $2d_k = k(N-k)/(N(N+2))$
- Operators $\psi_k(z)$ appears at OPE of disorder and order fields and have dimensions $\Delta_k = \frac{k(N-k)}{N}$.
- A dynamical symmetry is generated by $\psi = \psi_1$

$$(\zeta - z)^{\frac{2}{N}}\psi(z)\psi(\zeta) = (z - \zeta)^{\frac{2}{N}}\psi(\zeta)\psi(z)$$

- Matrix elements of disorder and order fields and their descendants can be found explicitely

Technical comment on going off-criticality

- Off-critical analogue of parafermionic algebra

$$z^{\frac{2}{N}}\frac{1}{1-\frac{w}{z}}\frac{(x^{-2}\frac{w}{z}|x^{2N})_\infty}{(x^{2+2N}\frac{w}{z}|x^{2N})_\infty}\Psi(z)\Psi(w)=$$
$$=w^{\frac{2}{N}}\frac{1}{1-\frac{z}{w}}\frac{(x^{-2}\frac{z}{w}|x^{2N})_\infty}{(x^{2+2N}\frac{z}{w}|x^{2N})_\infty}\Psi(w)\Psi(z)$$

The structure functions are not power functions but meromorphic ones

$$(z|q)_\infty=\prod_{j=0}^{\infty}(1-zq^j)$$

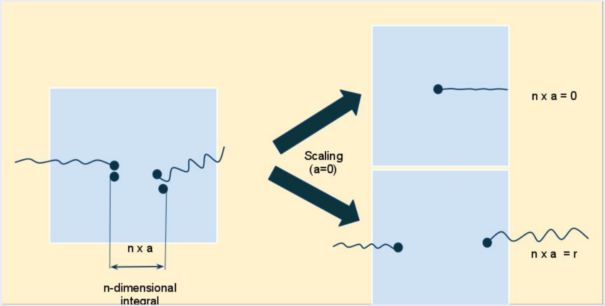
- In the conformal limit elliptic algebras become conformal ones $\Psi\to\psi$

$$\lim_{x\to 1}\frac{(x^{-2}\frac{w}{z}|x^{2N})_\infty}{(x^{2+2N}\frac{w}{z}|x^{2N})_\infty}=(1-\frac{w}{z})^{\frac{N+2}{N}}$$

Statements of off-critical model

- Deformed parafermionic algebra is a dynamical symmetry algebra of off-critical lattice model. Its irreps describe the lattice space of states in the Corner Transfer Matrix approach
- Disorder fieds and their parafermionic descendants, including order fields are deformed vertex operators acting in the irreps of this algebra
- Operators, diagonalizing the Transfer matrix and creating asymptotic states are Ψ_k. The scattering matrix of excitations become in the scaling limit the $S(\beta)$ matrix for massive QFT
- Lattice correlation functions and form factors can be exactly computed in terms of multiple q-Selberg type integrals

Alternatives in continuous limit



- Studying 1 point functions and form factors
- Handling multiple integrals aiming two point function

Results for scaling: Vacuum Expectation Values

Exact VEV is conformal normalization from deformed Vertex operators

$$\langle \mu_k \rangle = \left(\frac{\Gamma(2/N)\Gamma(1-1/N)}{\Gamma(1/N)} M \right)^{2d_k} \times$$

$$\exp \int_0^\infty \frac{dt}{t} \left(\frac{\sinh kt \sinh (N-k)t \cosh Nt}{\sinh Nt \sinh (N+2)t} - 2d_k e^{-2(N+2)t} \right)$$

$$\frac{\langle (\psi^\dagger)^l (\bar{\psi})^l \mu_k \rangle}{\langle \mu_k \rangle} = \left(M \frac{(N+2)\Gamma(2/N)\Gamma(1-1/N)}{N\Gamma(1/N)} \right)^{\frac{2l(k-l)}{N}}$$

$$\times \prod_{i=0}^{l-1} \frac{\Gamma(1+\frac{i+1}{N})\Gamma(1-\frac{k-i}{N})}{\Gamma(1+\frac{k-i}{N})\Gamma(1-\frac{i+1}{N})}$$

Results for scaling: Form factors

Maps in the space of form factors

- Multiparticle form factors are found exactly in terms of double-sine functions for

$$\langle \mu_k|\beta_1,...,\beta_n\rangle,\ \langle s_k|\beta_1,...,\beta_n\rangle,\ \langle \psi_k|\beta_1,...,\beta_n\rangle$$

- The algebraic maps in the space of form factors of scaling fields induced by the action of deformed parafermionic currents are proposed and checked

$$\langle \mu_k|\beta_1,...\rangle \longrightarrow \langle (\psi^\dagger)^l(\bar{\psi})^s\mu_k|\beta_1,...\rangle$$

- The first non-trivial null vector equations were checked

$$\langle \bar{\partial}\psi|\beta_1,\cdots\rangle = \langle \psi\varepsilon|\beta_1\cdots\rangle\,,$$
$$\langle W_{-2}\varepsilon|\beta_1,\cdots\rangle = \langle L_{-1}W_{-1}\varepsilon|\beta_1\cdots\rangle\,,$$

Correlation functions in scaling massive model

Long and short distance expansions

$$\langle \Phi(r)\Phi(0)\rangle = \begin{cases} \sum C_j(r,\lambda)\langle A_j\rangle, & Mr << 1 \\ \sum \int d\beta_1 \cdots \langle \Phi(r)|\beta_1,\cdots\rangle\langle\beta_1,\cdots|\Phi(0)\rangle, & Mr >> 1 \end{cases}$$

Necessary data for analysis at $Mr \sim 1$

- Correlators from CFT and perturbation theory for structure functions
- Matrix elements of scaling field Φ in basis of asymptotic states $|\beta_1,\beta_2...\rangle$, i.e. form factors $\langle\Phi|\beta,...\rangle$
- Relation betwen mass and coupling constant $\lambda \sim M^{2N/(N+2)}$
- Vacuum expectation values $\langle A_j\rangle$ for basis scaling fields

Results for massive QFT: Correlation functions

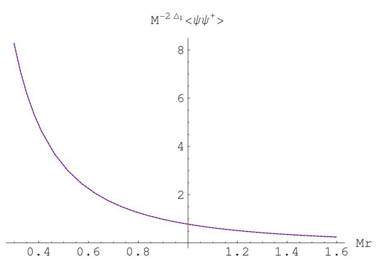
For correlation functions $\langle s_1(r)s_1^\dagger(0)\rangle$, $\langle \mu_1(r)\mu_1^\dagger(0)\rangle, \langle \psi_1(r)\psi_1^\dagger(0)\rangle$

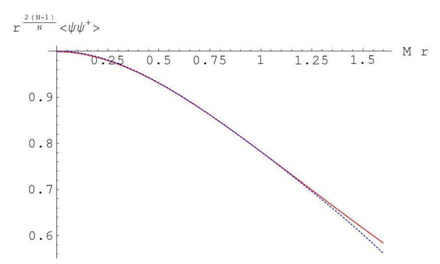
- UV: Conformal Perturbation Theory
- IR: Form factors spectral decomposition
- For N=2,3,4,... the numerical data are matched at intermediate distances

Correlation functions at $Mr \sim 1$

Example of parafermionic fields correlator in massive theory

Comparing short (CFT perturbation - blue) and long (form factor expansion - red) distance asymptotics at intermediate distances. Example N=9





Conclusion

Space of local fields in integrable perturbed CFT

We develop the algebraic approach by studying algebraic maps in the space of form factors. The recent results

- Confirmation of null vector equations for the form factors of Z_4 models in comparihson with second reflection less point in Sine-Gordon model. Studying resonances.
- General expression for null vectors in the space of form factors in perturbed minimal models