ON ab initio THEORY OF NUCLEAR PAIRING

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M. Baldo, U. Lombardo, S.S. Pankratov, and E.E. Saperstein, J. Phys. G: Nucl. Part. Phys. 37 (2010) 064016.

S. S. Pankratov, M. Baldo, U. Lombardo, E. E. Saperstein, and M.V. Zverev, JETP Lett., 92, 92 (2010).

E.E. Saperstein, M. Baldo, U. Lombardo, S.S. Pankratov, and M.V. Zverev, Phys. Phys. Rev. C., to be publ. (2011).

Up to now there is no consistent ab initio theory of nuclear matter.

The Brueckner theory is a good first step, but next steps include various complicated many-body theory corrections which now can not be taken into account in an unambiguous way.

Why the idea of the ab initio theory of nuclear pairing is not absolutely unreasonable?

Although a finite nucleus is much more complicated system than infinite matter, there are simplifications for some phenomena (including pairing).

The point is in the <u>surface nature of pairing in nuclei</u>: the effective pairing interaction at the surface is much stronger (up to 10 times) than that inside. But at the surface the Brueckner theory is valid trivially.

$$F^{\xi} = C_0[\, f_{ex}^{\xi} + (f_{in}^{\xi} - f_{ex}^{\xi})\rho / \rho_0 \,],$$

$$| f_{in}^{\xi} / f_{ex}^{\xi} | \cong 0.1$$

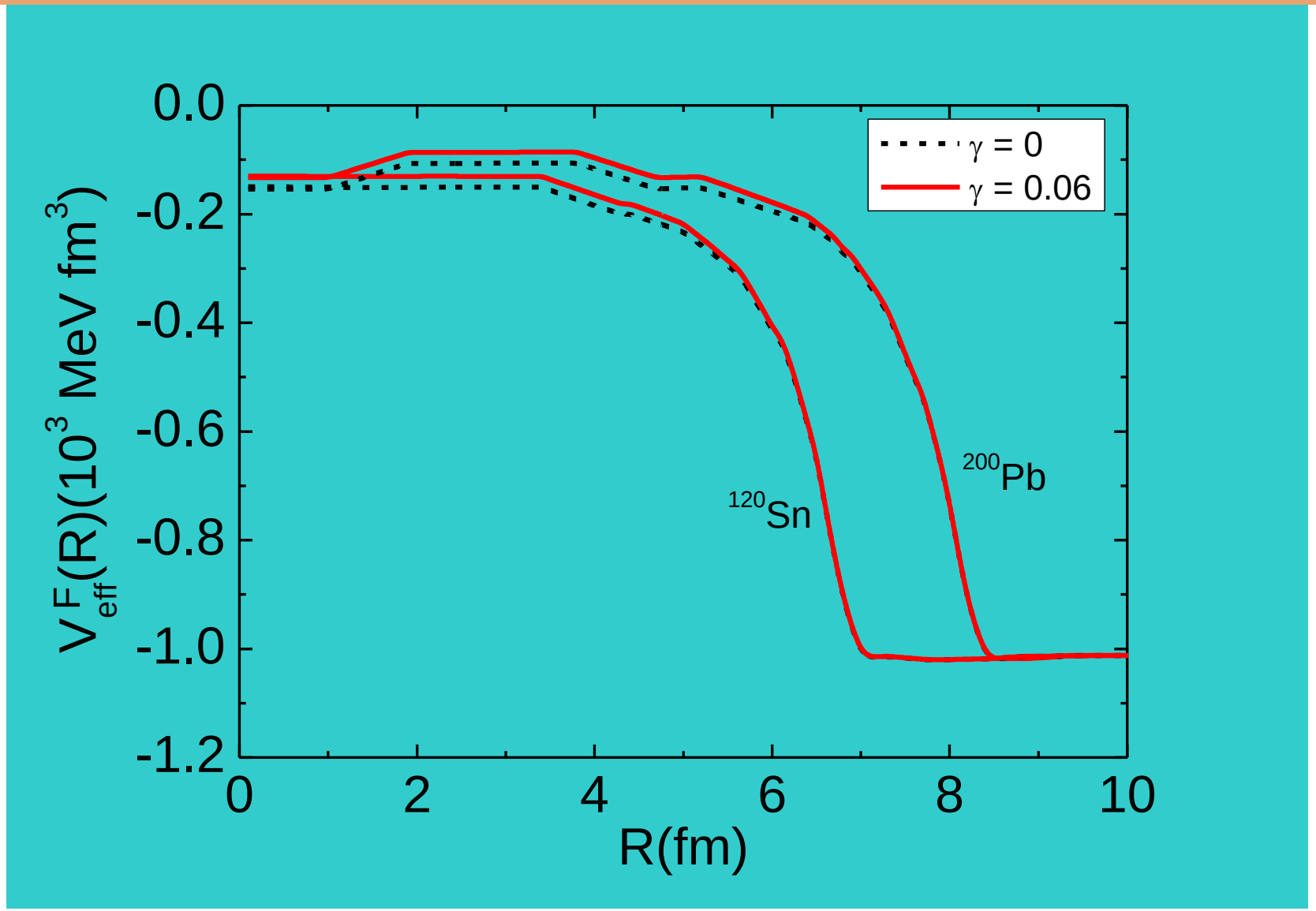
Why the ab initio theory of pairing is important?

The ab initio effective pairing interaction is strongly μ-dependent. It is important nearby the drip-line: $\mu \to 0$

All predictions for the drip-line based on pure phenomenological force are doubtful.

M. Baldo, U. Lombardo, E.E. Saperstein, M.V. Zverev, Phys.Lett. B533, 17 (2002)

Localized effective pairing interaction in 200Pb and 120Sn (model space with $E_0 = 40MeV$)



Idea: to find V_{eff} at the surface from the first principles, with the HOPE that some inaccuracy in the small inner value is not important.

Failed because of the exponential dependence of the gap on the interaction strength: in the weak coupling limit of the BCS approach

$$\Delta_F \approx 2\varepsilon_F \exp(1/\nu_F V_{eff})$$

where

$$\nu_F = m^* k_F / \pi^2$$

is the density of state at the Fermi surface :

Small change of V_{eff} results in significant variation of Δ_F

General many-body theory gap equation

$$\Delta = -uGG^s\Delta$$

u is the interaction block irreducible in the pp-channel, G^s (G) - Green function with (without) pairing. BCS approximation: $u \to V$ (V - free NN-potential) : Bogolyubov gap equation:

$$\Delta = -V\kappa$$

- $$\kappa = \int \frac{d\varepsilon}{2\pi i} GG^s\Delta$$ the anomalous density matrix:

$$\kappa(r_1, r_2) = \sum u_i(r_1)v_i(r_2)$$

- Very slow convergence in the λ-basis

Milan group:

1. F. Barranco, R. A. Broglia, H. Esbensen, and E. Vigezzi, Phys. Lett. B390, 13 (1997).

Argonne v14 NN-potential + Saxon-Woods potential, ^{120}Sn $(E_{\max} = 600 MeV)$

$\Delta_{BCS} \approx 2 MeV$ **whereas** $\Delta_{\exp} \approx 1.3 MeV$

It was shown at the first time that the ab initio BCS gap equation is relevant and is a good starting point

2. F. Barranco, R.A. Broglia, G. Colo, et al., Eur. Phys. J. A **21**, 57 (2004).

3. A. Pastore, F. Barranco, R.A. Broglia, and E. Vigezzi, Phys. Rev. C **78**, 024315 (2008).

Argonne v14 NN-potential + SLy4 Skyrme basis $(m^* \approx 0.7m)$, the same nucl. $(E_{\max} = 800MeV)$

$\Delta_{BCS} \approx 1MeV$; $\Delta_{\exp} \approx 1.3MeV$

The difference was explained with corrections to the BCS approximation (polarization diagrams in the interaction block U, Z-factor and other corrections to Green functions…), the sum of them $\approx 0.4MeV$

- A blow-up in 2008: T. Duguet et al.

T. Duguet and T. Lesinski, Eur. Phys. J. Special Topics **156,** 207 (2008).

K. Hebeler, T. Duguet, T. Lesinski, and A. Schwenk, Phys. Rev. C **80**, 044321 (2009).

'Low-k' NN-potential $(E_{\max} \approx 300 MeV)$ + SLy4 basis

A lot of nuclei (n,p), for 120 Sn $\Delta_{BCS} = 1.6 MeV$

We analyzed reasons of the contradiction

S. S. Pankratov, M. Baldo, U. Lombardo, E. E. Saperstein, M. V. Zverev, S.V. Tolokonnikov, JETP Lett. **90**, 560 (2009)

In fact, different k-dependence of m*:

Should coincide if in Milan calculations m*=0.7m for

E <300 MeV and m*=m for E>300 MeV

To avoid the slow convergence problem, we use a 2-step method: the Hilbert space $S = S_0 + S'$

$$\Delta_{BCS} = -V_{eff}^{BCS} \kappa_0$$

$$\kappa_0 = \sum u_i v_i, (\lambda, \lambda') \in S_0$$

$$V_{eff}^{BCS} = V + VGGV_{eff}^{BCS}, (\lambda, \lambda') \in S'$$

In the S' space 'Local Potential Approximation' (LPA):

At each point R, Bethe-Goldstone eq. for the effective interaction is solved in inf. nuclear matter imbedded into the potential well U(R) (with m*=m).

Accuracy of the LPA is several percent even at the surface (contrary to the LDA).

Instead of attempts to calculate in-volume corrections to the BCS scheme (m*(r,k), induced interaction,…) we suggest to try a semi-microscopic model

$$V_{eff}(r_i) = V_{eff}^{BCS}(r_i) + C_0\gamma\frac{\rho(r_1)}{\rho(0)}\delta(r_1 - r_2)\delta(r_1 - r_3)\delta(r_2 - r_4)$$

with one (small) phenomenological parameter γ ($C_0 = 300 Mev \bullet fm^3$) $\rho(r)$ - nucleon density of the type under consideration . In the model space, EDF (S.A. Fayans et al.) basis (m*=m) is used

Optimal $\gamma = 0.06$: $\delta\Delta_{rms} \approx 0.13 MeV$ for 34 nuclei (25 neutron pairing, 9 proton pairing)

Numerical details:

Argonne v18 NN-potential, basis of the EDF method by S.A. Fayans et al. (DF3 functional, $m^* \equiv m$!!) ,

Spherical box with R=16 fm, h=0.05 fm, model space S_0 with $E_0 = 40 MeV, S'$ with $E_{\max} = 1000 MeV$

Milan: R=15 fm, h=0.1 fm, $E_{\max} = 800 MeV$, v14 force, SLy4 basis

Duguet et al.: R=30 fm, h=0.3 fm, $E_{\max} \approx 300 MeV$

'Low-k' force, SLy4 basis

“Experimental gap”

Recipe (came from the model Δ=const):

$$2\Delta^{+}_{\exp} = \delta^{+}_{2} M = 2M(A+1) - M(A+2) - M(A),$$

$$2\Delta^{-}_{\exp} = \delta^{-}_{2} M = 2M(A-1) - M(A-2) - M(A),$$

$$2\Delta_{\exp} = \delta_{2} M = (\delta^{+}_{2} + \delta^{-}_{2})/2.$$

Is compared with Δ_F - the diagonal $\Delta_{\lambda\lambda}$ matrix element of the gap averaged over several levels nearby the Fermi surface.

Accuracy? – “Theoretical experiment”: For the self-consistent EDF method by S.A. Fayans et al.

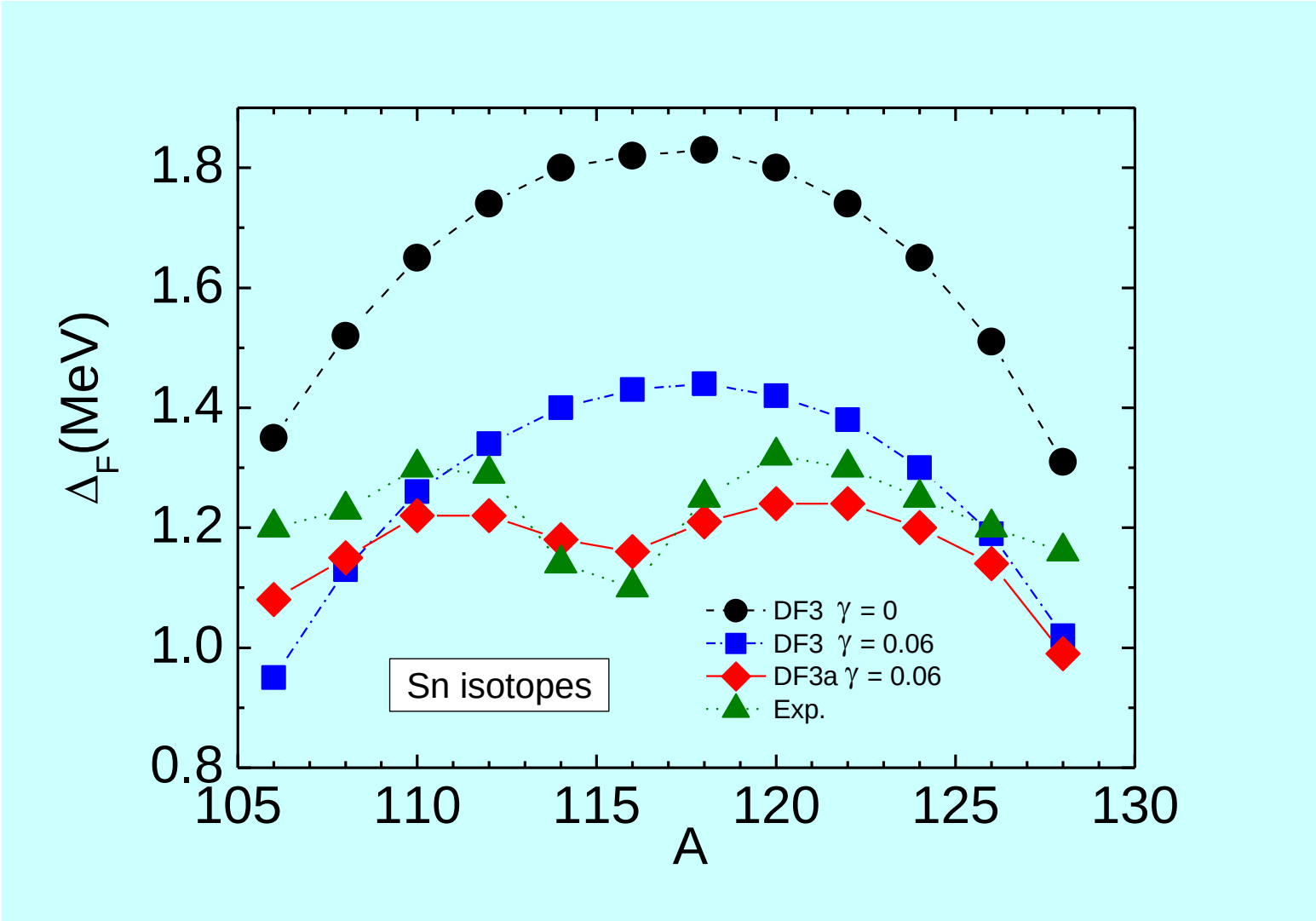
'Theoretical' gap:

$$\Delta_F = \sum_\lambda (2j+1)\Delta_{\lambda\lambda} \Big/ \sum_\lambda (2j+1),$$

$$|\varepsilon_\lambda - \mu| < 3MeV$$

Accuracy of the recipe for experimental gap of the order of 0.1-0.2 MeV (see a theoretical experiment below), therefore no correction to the particle number nonconservation effects is not made

Role of the s.p. spectrum: DF3 (S. Fayans et al.) versus DF3a (s-l and tensor terms are modified)



Spin-orbit and 1-st Landau-Migdal spin harmonic (effective tensor force) terms of the EDF

$$F_{sl} = C_0 r_0^2 (\kappa + \kappa' \tau_1 \tau_2)[\nabla_1 \delta(r_1 - r_2) \times (p_1 - p_2)] \bullet (\sigma_1 + \sigma_2)$$

$$F_s = C_0 r_0^2 (g_1 + g_1' \tau_1 \tau_2) \delta(r_1 - r_2)(p_1 p_2)(\sigma_1 \sigma_2)$$

$$\rho_{sl}(r) = \sum_\lambda n_\lambda < \psi_\lambda^*(r)(\sigma l)\psi_\lambda(r) >$$

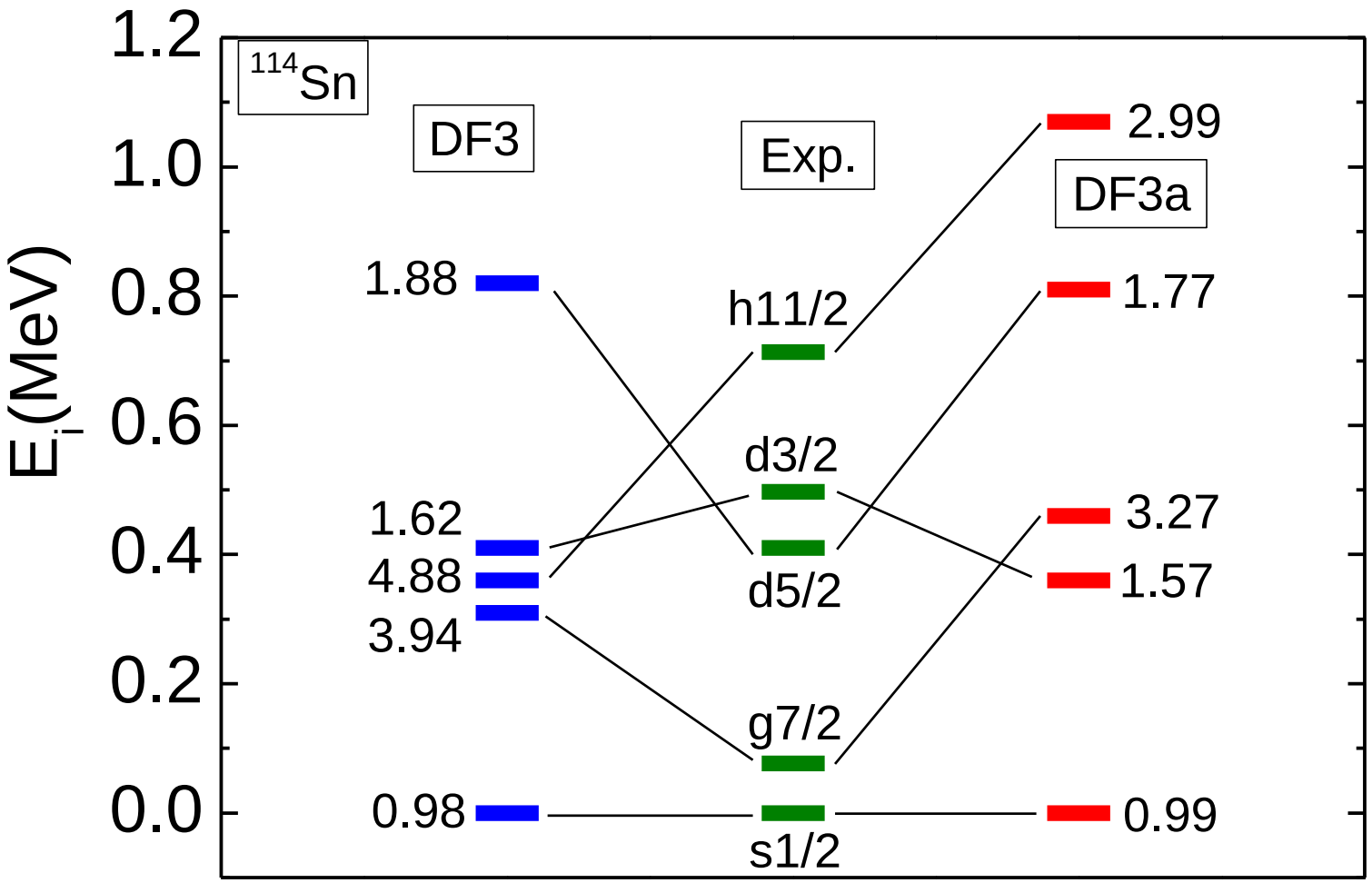
$$\kappa, \kappa', g_1, g_1' \quad C_0 = 300 MeV \bullet fm^3, r_0 = 1.147\ fm$$

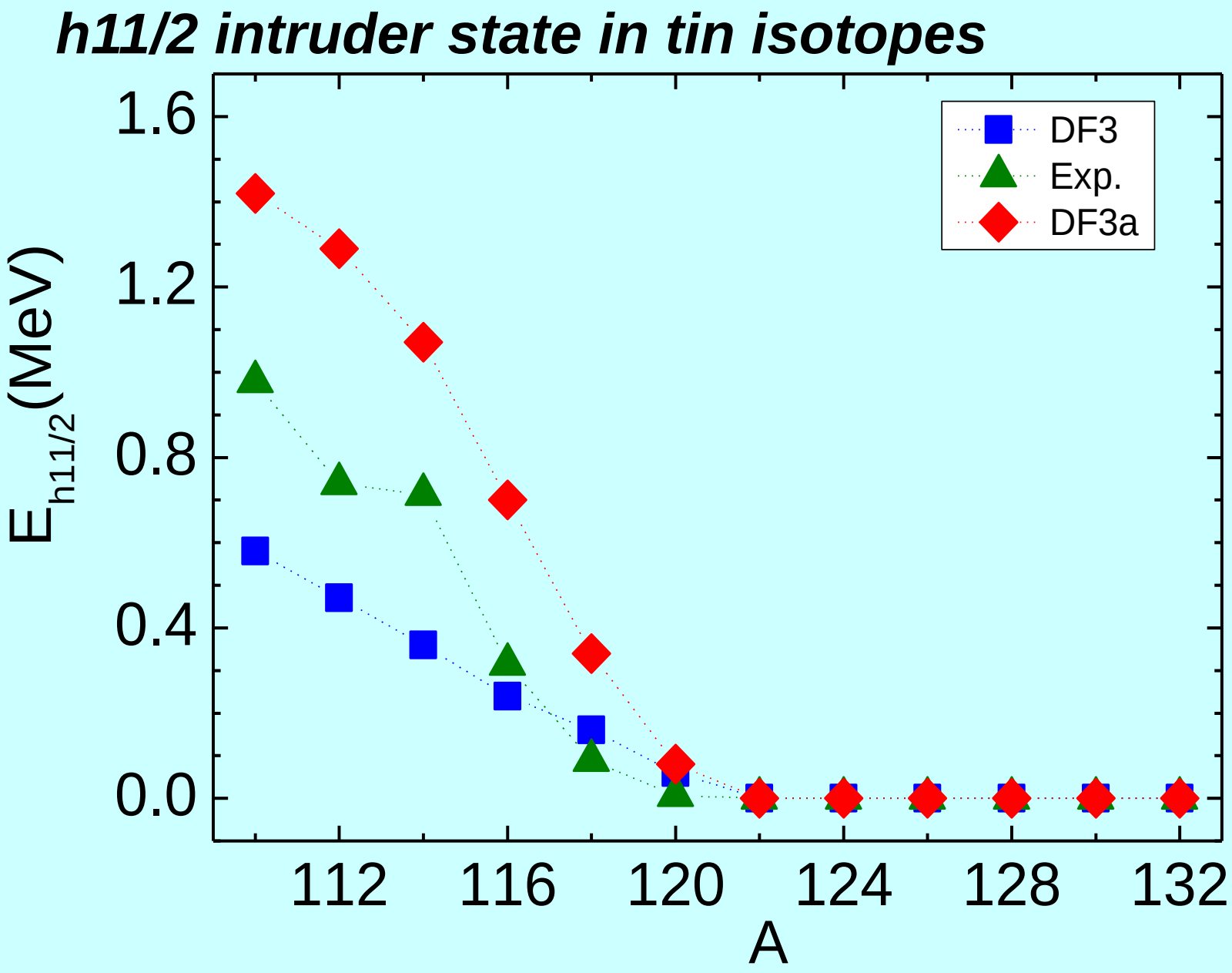
$$DF3: \kappa = 0.21, g_1' = -0.12$$

$$DF3a: \kappa = 0.185, g_1' = -0.3$$

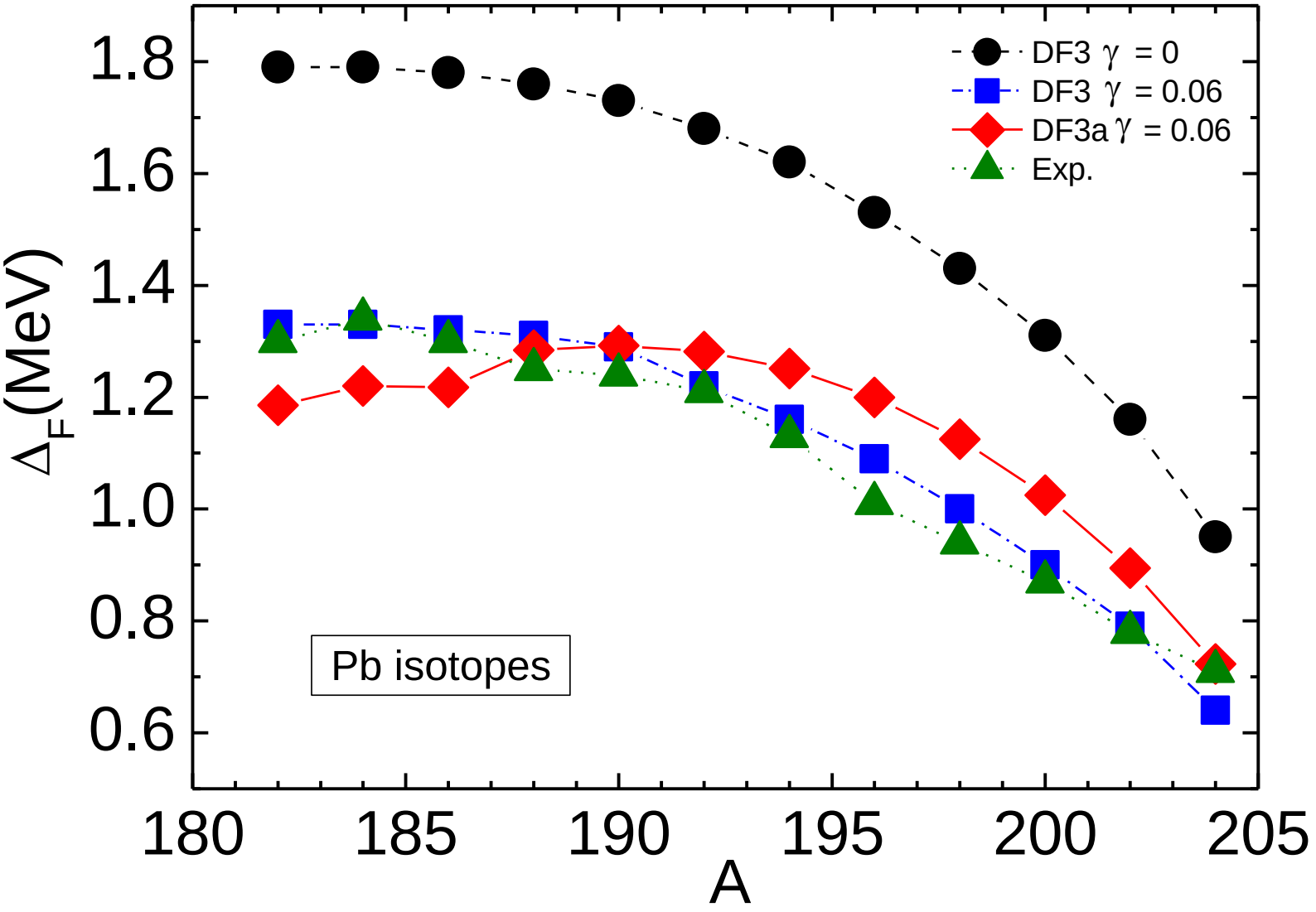
S.V. Tolokonnikov, E.E. Saperstein, Phys. At. Nucl. 73, 1684 (2010).

s.p. levels in 114 Sn




h11/2 intruder state in tin isotopes
DF3
Exp.
DF3a
E_h11/2(MeV)
1.6
1.2
0.8
0.4
0.0
112
116
120
124
128
132
A

s.p. levels in 116 Sn


DF3 γ = 0
DF3 γ = 0.06
DF3a γ = 0.06
Exp.
Pb isotopes
Δ_F(MeV)
A
1.8
1.6
1.4
1.2
1.0
0.8
0.6
180
185
190
195
200
205

A (Pb)	Δ_F^n			$\Delta_{\exp}$
	$\gamma = 0$	0.06	0.08	
180	1.79	1.33	1.20	1.34
182	1.79	1.33	1.20	1.30
186	1.78	1.32	1.19	1.30
188	1.76	1.31	1.17	1.25
190	1.73	1.29	1.16	1.24
192	1.68	1.22	1.09	1.21
194	1.62	1.16	1.03	1.13
196	1.53	1.09	0.96	1.01
198	1.43	1.00	0.87	0.94
200	1.31	0.90	0.80	0.87
202	1.16	0.79	0.69	0.78

	γ = 0	0.06	0.08	exp
204 Pb	0.95	0.64	0.56	0.71
106 Sn	1.35	0.95	0.83	1.20
108	1.52	1.13	1.01	1.23
110	1.65	1.26	1.14	1.30
112	1.74	1.34	1.23	1.29
114	1.80	1.40	1.28	1.14
116	1.82	1.43	1.31	1.10
118	1.83	1.44	1.32	1.25
120	1.80	1.42	1.31	1.32
122	1.74	1.38	1.28	1.30
124	1.65	1.30	1.21	1.25

	$\gamma = 0$	0.06	0.08	exp
126 Sn	1.51	1.19	1.10	1.20
128 Sn	1.31	1.02	0.94	1.16

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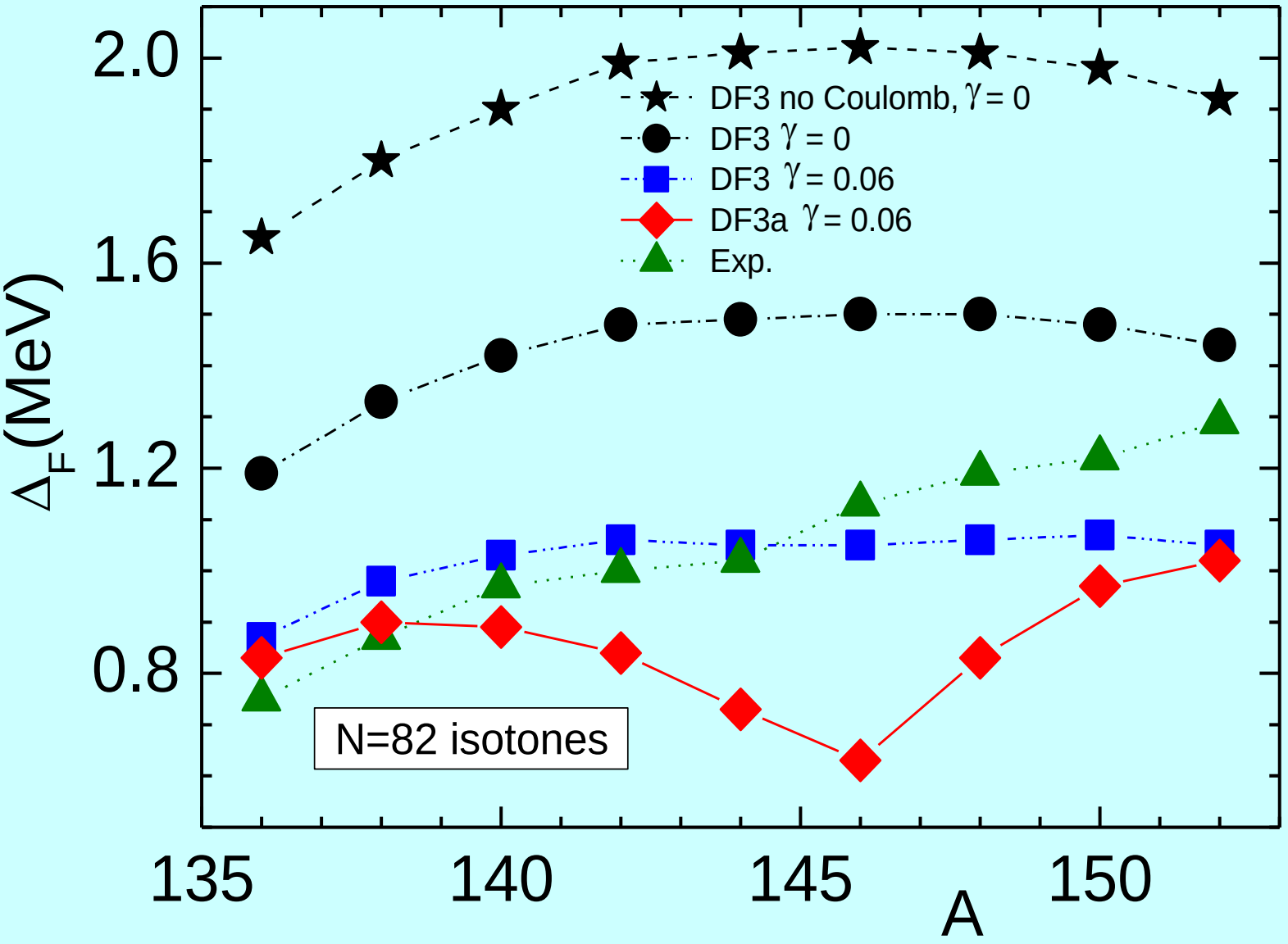
44 Ca	1.83	1.50	1.41	1.54

Proton pairing (with Coulomb interaction added)



The Coulomb interaction in the pairing problem is not renormalized with the strong interaction:

$V_{p,eff}^{BCS} = V_{n,eff}^{BCS} + V_C$, where V_C is bare:

Situation is similar to the Hartree term of the mean field, not to the Fock one. It contributes mainly to diagonal matrix elements

$< \lambda_1 \lambda_2 | V_C | \lambda_3 \lambda_4 >, \quad with \quad \lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda$

$\approx e^2 / R \approx 10 - 15\,\%$ of the main (strong) term

But contributes to the gap significantly,
Duguet et al. With Coulomb int. taken into account

$\gamma_p = \gamma_n = \gamma$

CONCLUSIONS

A simple semi-microscopic model for the pairing effective interaction:

$V_{eff} = V_{eff}^{BCS}$ (*ab initio*) + small in-volume phenomenological addendum works reasonably well in semi-magic nuclei (with accuracy of 0.1-0.2 MeV).

Results are very sensitive to the single-particle spectrum of the basis used in the gap equation, especially to position of high *j*-levels.

For the EDF *DF3* by S. Fayans et al.

$$\delta\Delta_{rms} \approx 0.13 MeV$$

for 34 nuclei (neutron and proton pairing), for *DF3a* approximately the same

Acuracy is essentially better than in the calculation by Duguet et al.

In the latter there is no explicit parameter,

but, in fact, there is an implicit one, a special kind of $m^*(k)$ function!