Universe in a helium droplet

topological Fermi liquids: from Migdal jump to topological Khodel fermion condensate

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Migdal-100, Chernogolovka June 25, 2011

* Topological media as topological objects in ***p***-space

* Fermi surface as vortex in ***p***-space

* Topological protection of Migdal jump and other singularities on Fermi surface

* Khodel Fermi condensate as π-vortex

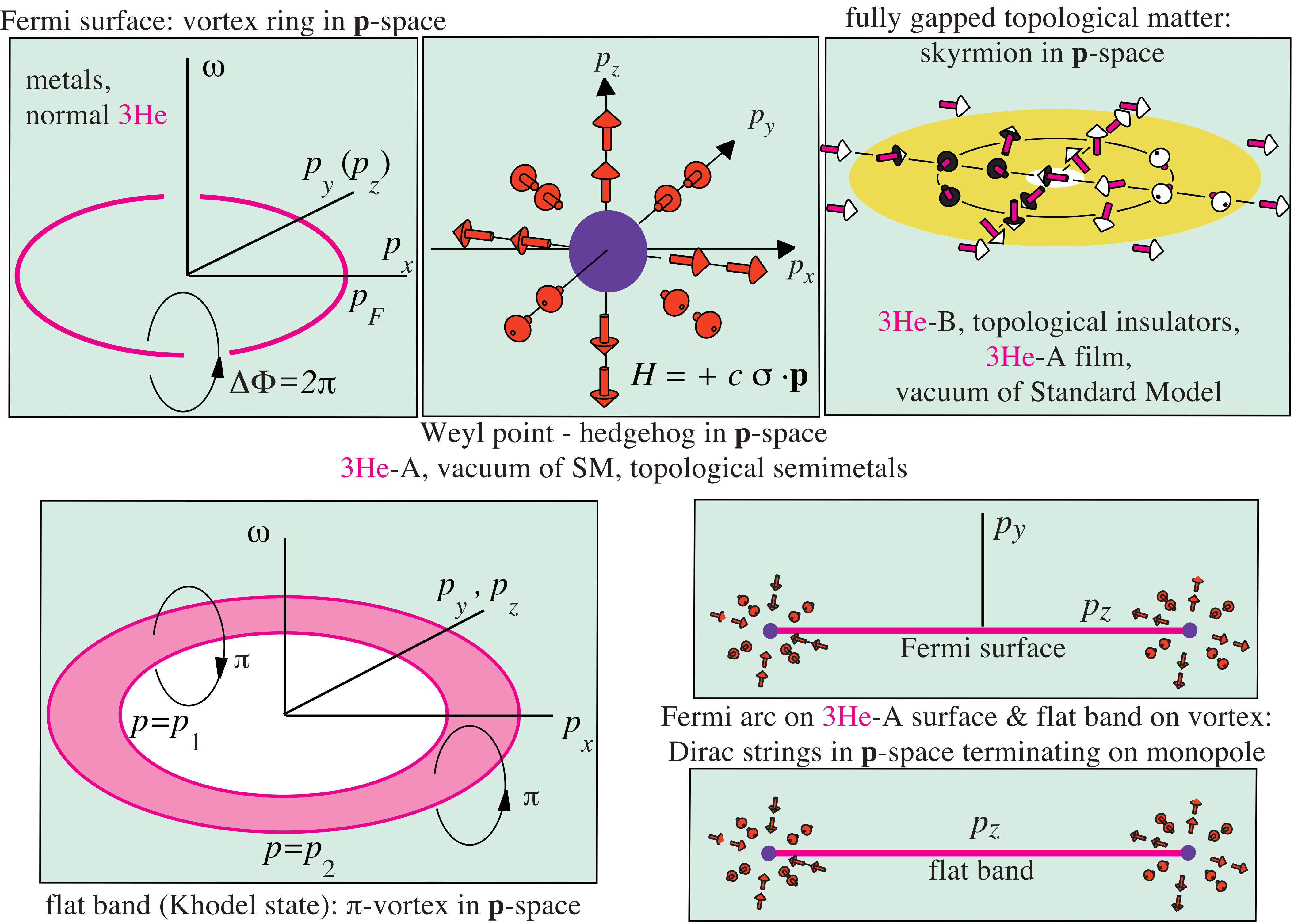
* Bulk-surface & bulk-vortex correspondence in topological matter

* New life of Fermi condensate: Topologically protected flat bands

* Can Fermi surface terminate? Fermi-arc in topological matter with Weyl points

* 3He Universe

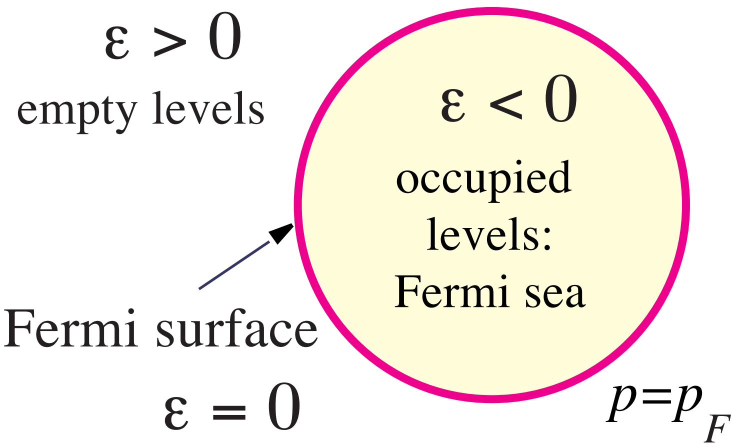
classes of topological matter as momentum-space objects



Topological stability of Fermi surface

Energy spectrum of
non-interacting gas of fermionic atoms

$$\varepsilon(p) \;=\; \frac{p^2}{2m} - \mu \;=\; \frac{p^2}{2m} - \frac{p_F^{\,2}}{2m}$$

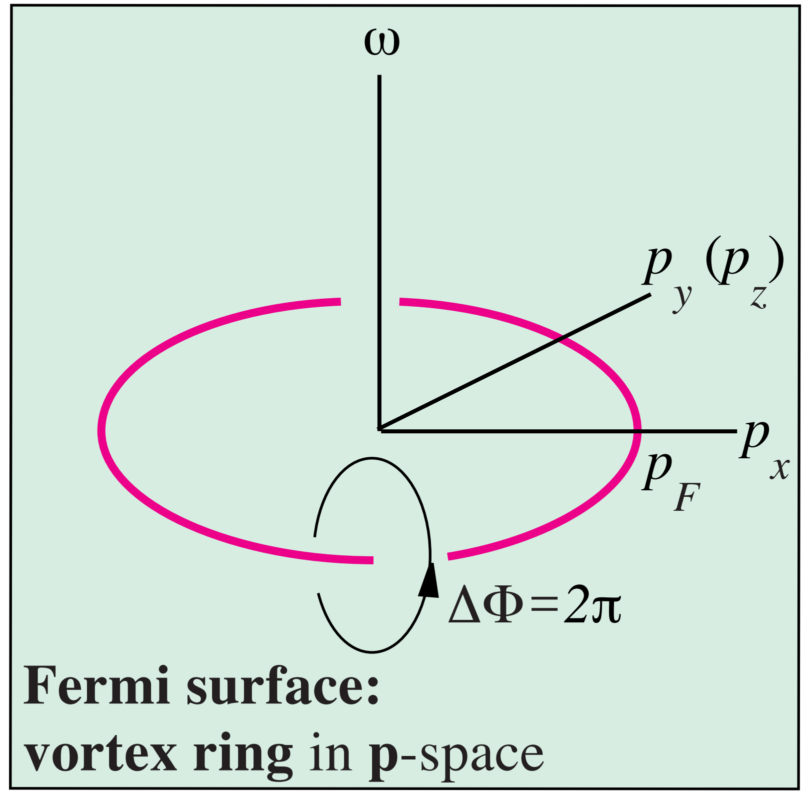


is Fermi surface a domain wall in momentum space?

no!
it is a **vortex ring**

Green's function

$$G^{-1} = i\omega - \varepsilon(p)$$

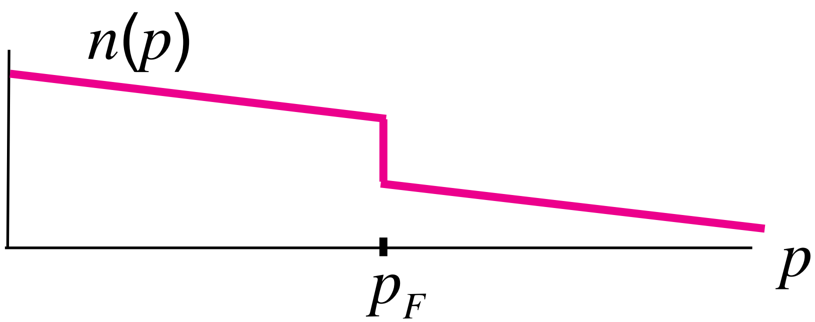


phase of Green's function
$G(\omega,\mathbf{p}) = |G| e^{\,i\Phi}$
has winding number $N = 1$

Migdal jump & p-space topology

* Singularity at Fermi surface is robust to perturbations:
winding number $N=1$ *cannot change continuously,*
interaction (perturbative) cannot destroy singularity

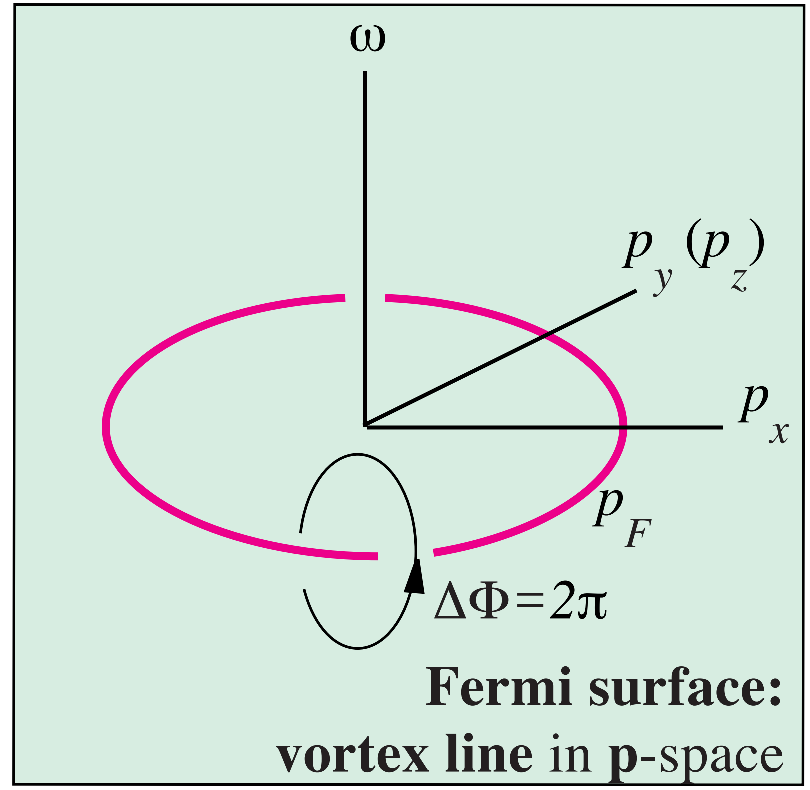
* Typical singularity: Migdal jump



* Other types of singularity: Luttinger Fermi liquid, marginal Fermi liquid, pseudo-gap ...

$$G(\omega,\mathbf{p}) = \frac{Z(p,\omega)}{i\omega - \varepsilon(p)}$$

$$Z(p,\omega) = (\omega^2 + \varepsilon^2(p))^\gamma$$



$$G(\omega,\mathbf{p})=|G|e^{\,i\Phi}$$

* Zeroes in Green's function instead of poles (for $\gamma > 1/2$) have the same winding number $N=1$

From Migdal jump to Landau Fermi-liquid

* Typical singularity in Fermi liquid: Migdal jump

* Quasiparticles in Fermi liquid as in Fermi gas

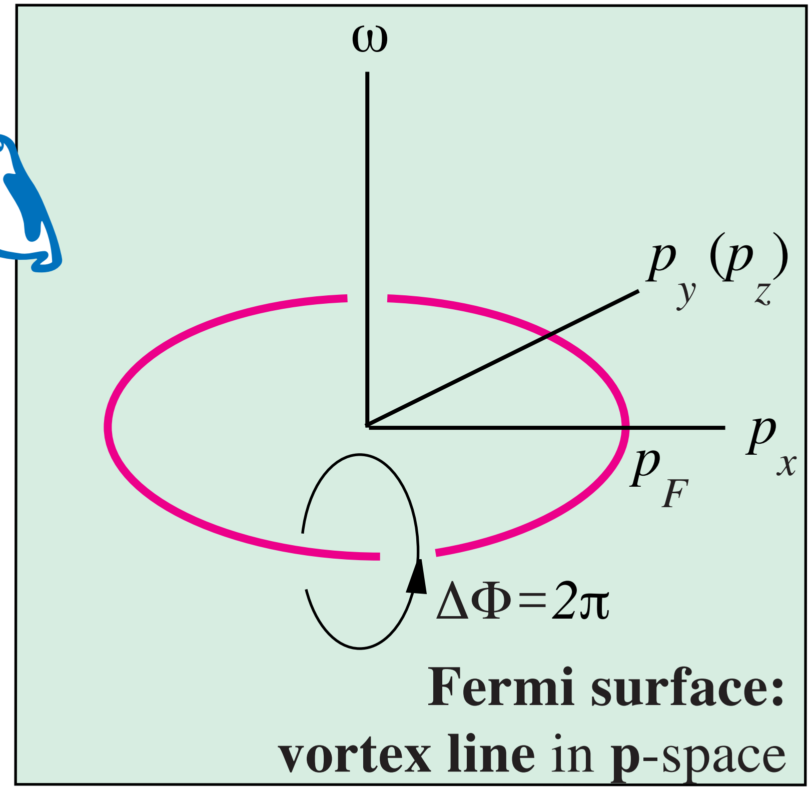
Landau theory of Fermi liquid
is topologically protected & thus is universal

all metals have Fermi surface ...

Not only metals.
Some superconductore too!

* Fermi surface in superconductors

"Stability conditions & Fermi surface topologies in a superconductor"
Gubankova-Schmitt-Wilczek, *Phys.Rev*. **B**74 (2006) 064505

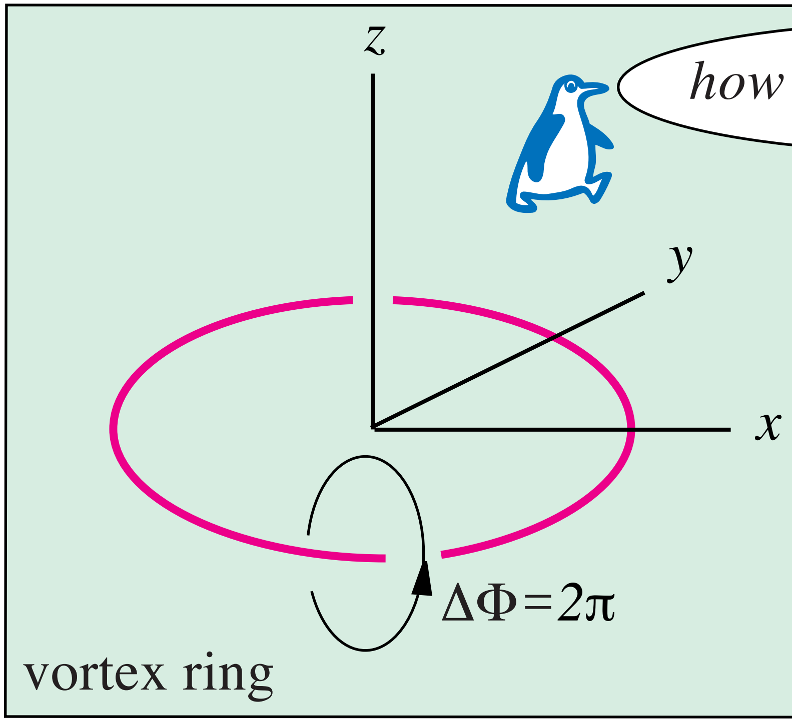


$$G(\omega,\mathbf{p})=|G|e^{i\Phi}$$

quantized vortex in **r**-space $\equiv$ **Fermi surface** in **p**-space

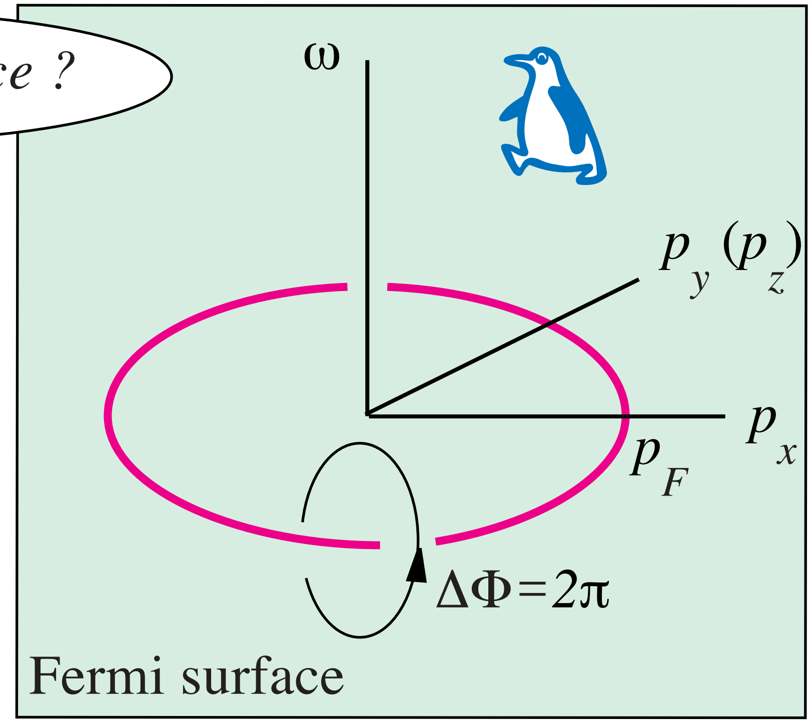
homotopy group π_1

Topology in **r**-space



how is it in **p**-*space ?*

Topology in **p**-space



winding number $N_1 = 1$

$$\Psi(\mathbf{r})=|\Psi|\, e^{i\Phi}$$

scalar order parameter of superfluid & superconductor

classes of mapping $S^1 \to U(1)$

manifold of broken symmetry vacuum states

$$G(\omega,\mathbf{p})=|G|e^{i\Phi}$$

Green's function (propagator)

classes of mapping $S^1 \to GL(\mathrm{n,C})$

space of non-degenerate complex matrices

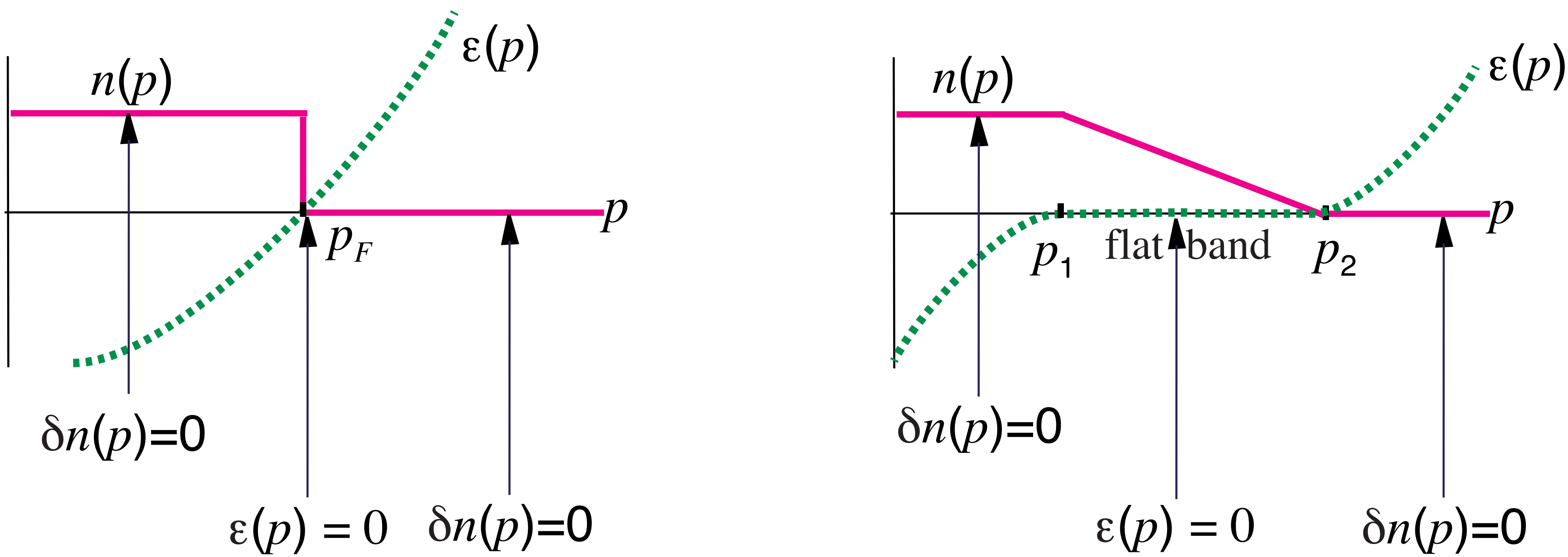
non-topological flat bands due to interaction

Khodel-Shaginyan fermion condensate

JETP Lett. **51**, 553 (1990)
GV, JETP Lett. **53**, 222 (1991)
Nozieres, J. Phys. (Fr.) **2**, 443 (1992)

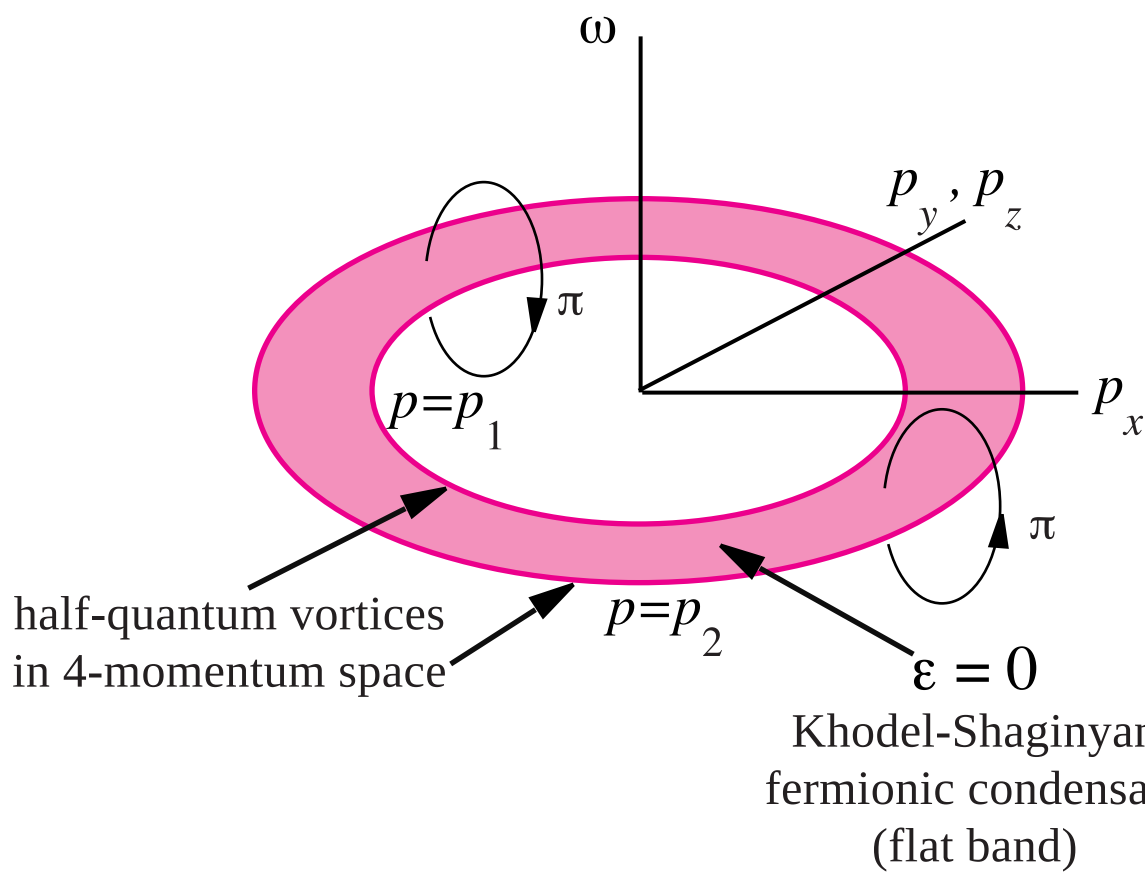
$$E\{n(p)\} \qquad \delta E\{n(p)\} = \int \varepsilon(p)\,\delta n(p)\, d^{\mathrm{d}}p = 0$$

solutions: $\varepsilon(p) = 0$ or $\delta n(p)=0$



splitting of Fermi surface to flat band

Flat band as momentum-space dark soliton terminated by half-quantum vortices



phase of Green's function changes by π across the "dark soliton"

topological correspondence:
topology in bulk protects gapless fermions on the surface or in vortex core

bulk-surface correspondence:

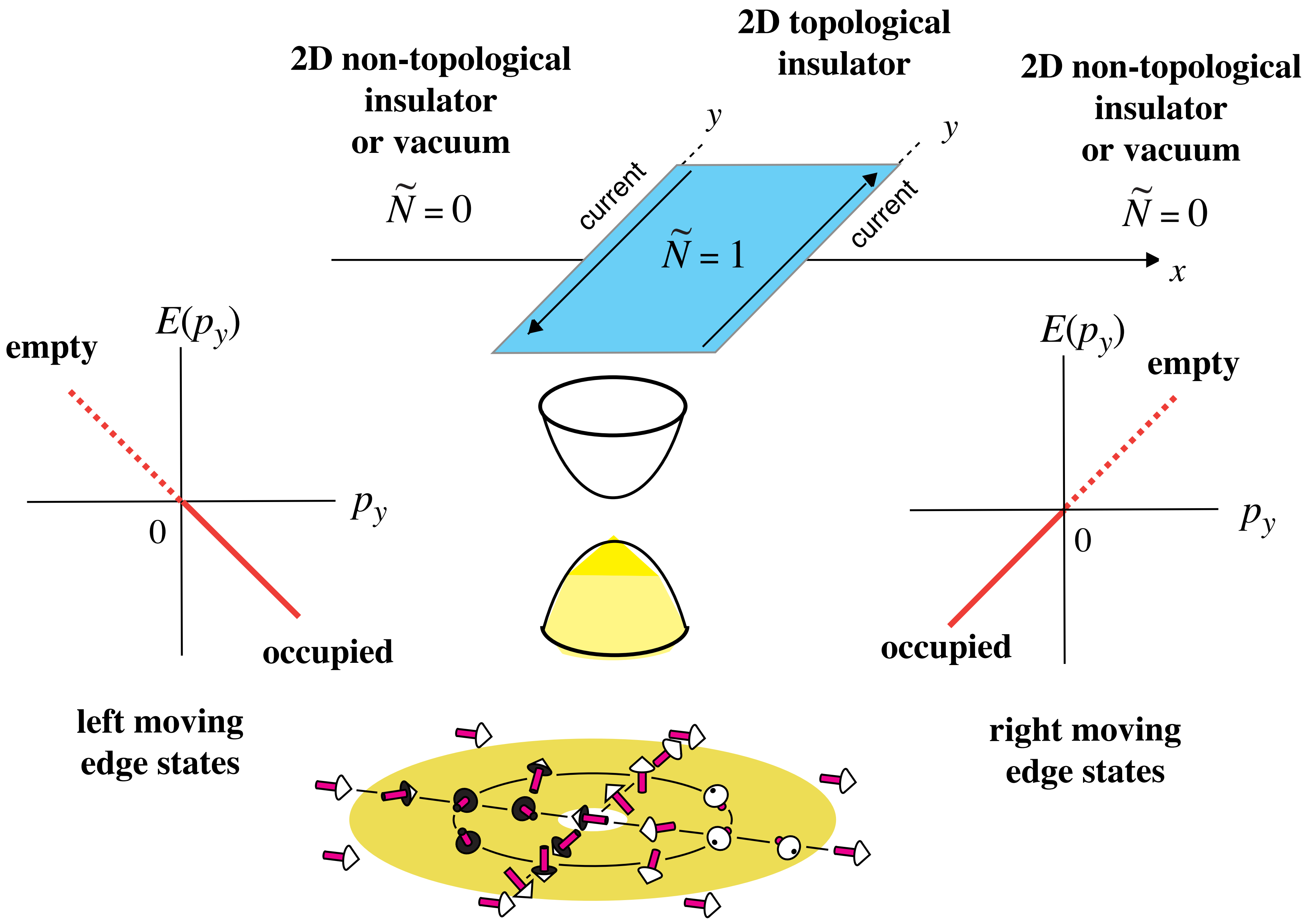
2D Quantum Hall insulator & 3He-A film	chiral edge states
3D topological insulator	Dirac fermions on surface
superfluid 3He-B	Majorana fermions on surface
superfluid 3He-A, Weyl point semimetal	Fermi arc on surface
graphene	dispersionless 1D flat band on surface
semimetal with Fermi lines	2D flat band on the surface

bulk-vortex correspondence:

superfluid 3He-A	1D flat band of zero modes in the core

Bulk-surface correspondence:

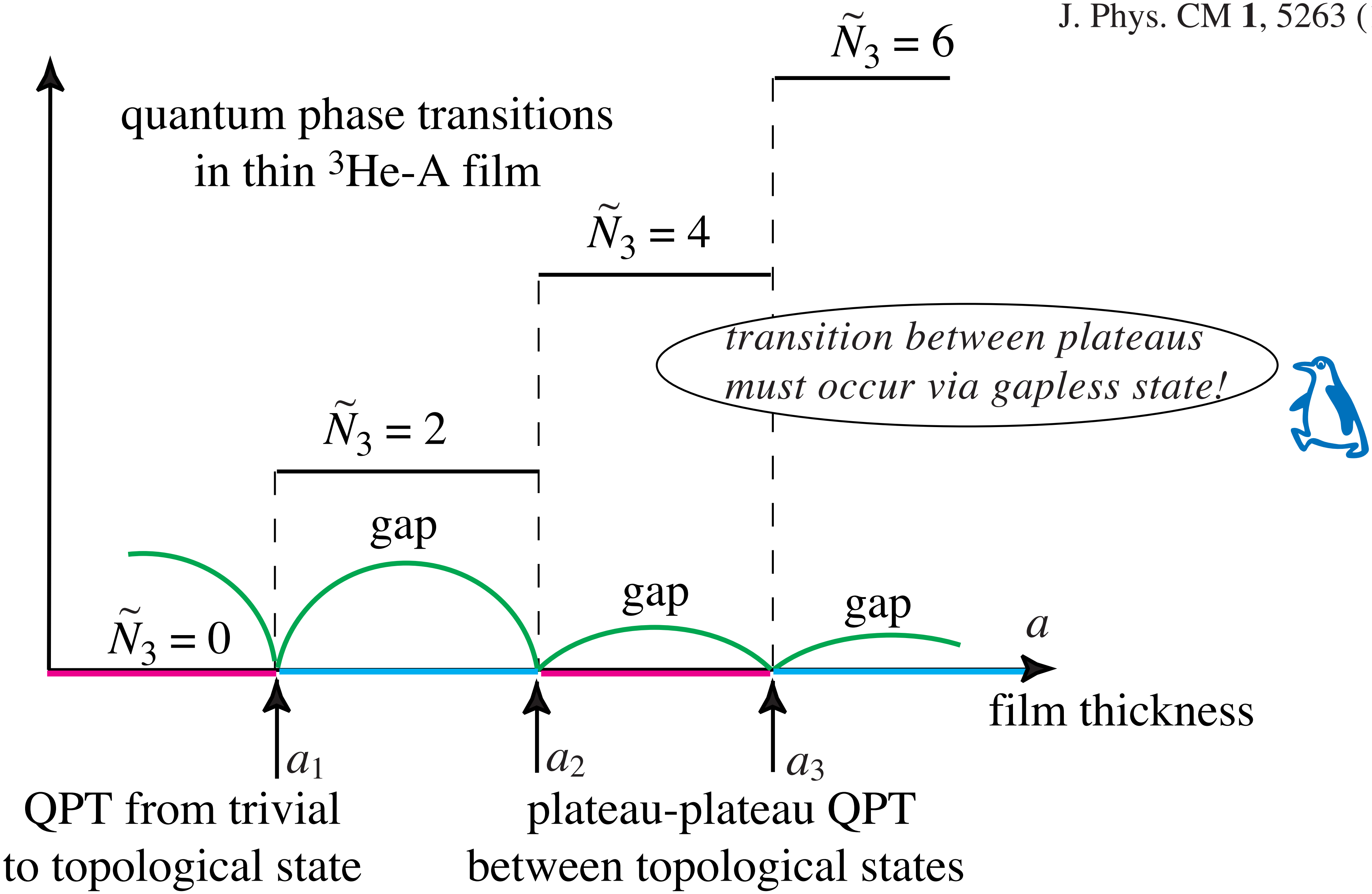
Edge states in quantum Hall topological insulators & 3He-A film



p-space invariant in terms of Green's function & topological QPT

$$\boxed{\tilde{N}_3 = \frac{1}{24\pi^2}\, e_{\mu\nu\lambda}\, \mathbf{tr} \int d^2p\, d\omega\, \mathbf{G}\, \partial^{\mu}\, \mathbf{G}^{-1}\, \mathbf{G}\, \partial^{\nu}\, \mathbf{G}^{-1} \mathbf{G}\, \partial^{\lambda}\, \mathbf{G}^{-1}}$$

GV & Yakovenko
J. Phys. CM **1**, 5263 (1989)



topology of graphene nodes

$$\boxed{N = \frac{1}{4\pi i}\,\mathrm{tr}\left[\mathbf{K}\oint dl\, \mathbf{H}^{-1}\, \boldsymbol{\partial}_l\, \mathbf{H}\right]}$$

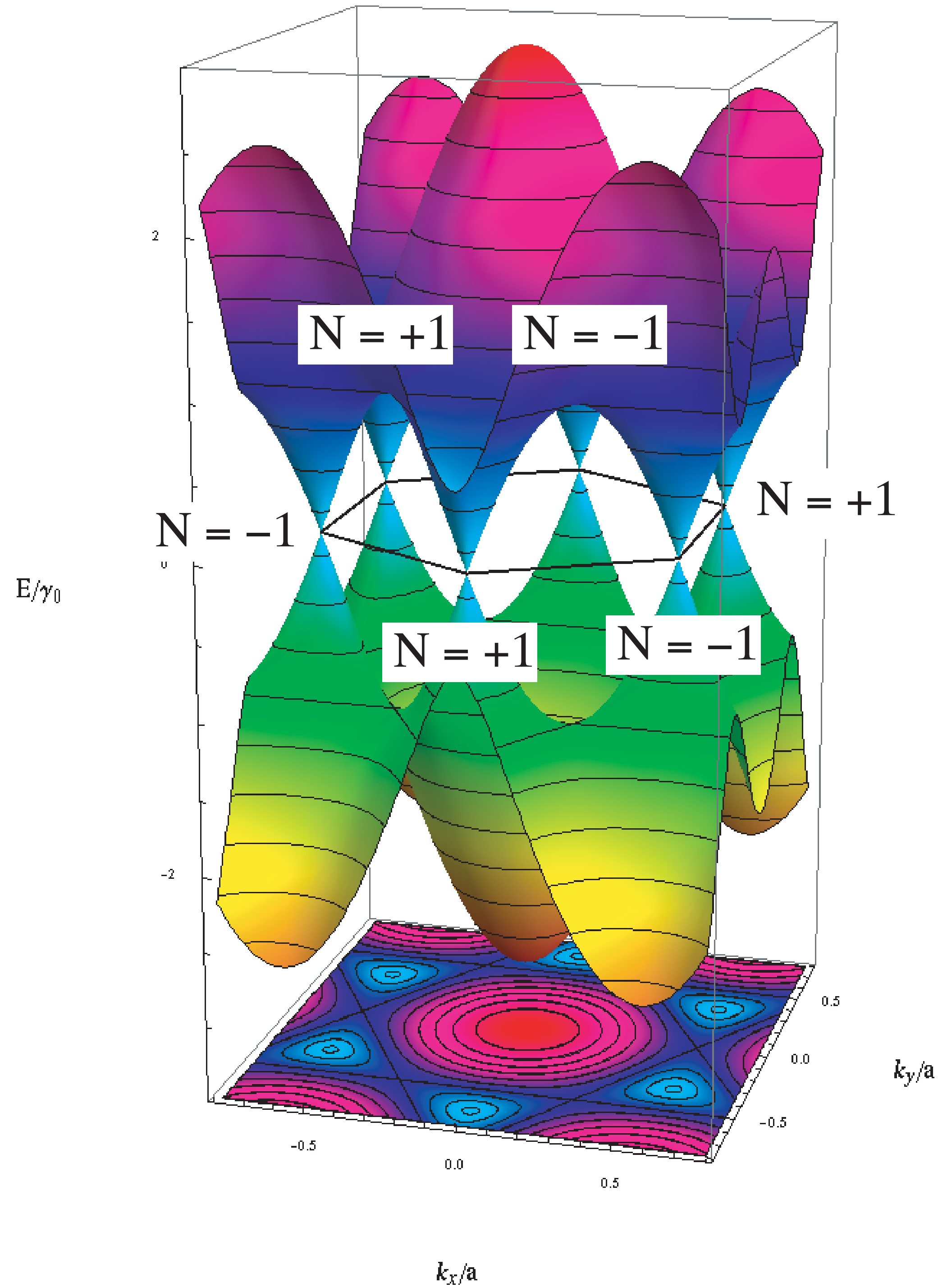
K - symmetry operator,
commuting or anti-commuting with **H**

close to nodes:

$$\mathbf{H}_{N=+1} = \tau_x p_x + \tau_y p_y$$

$$\mathbf{H}_{N=-1} = \tau_x p_x - \tau_y p_y$$

$$\mathbf{K} = \tau_z$$

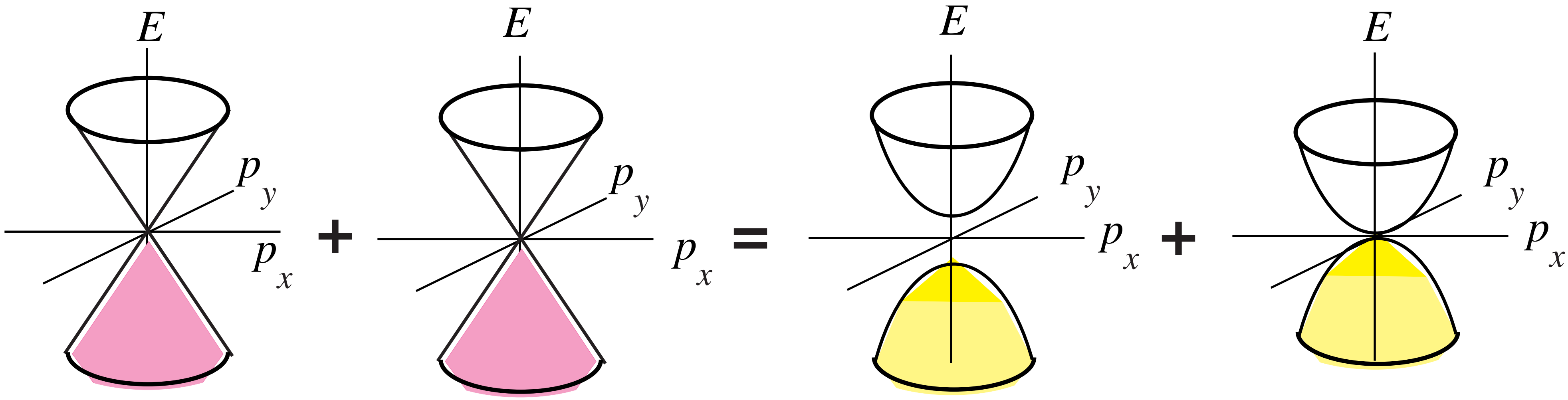


exotic fermions:
massless fermions with quardatic dispersion
semi-Dirac fermions
fermions with cubic and quartic dispersion

bilayer graphene
double cuprate layer
surface of top. insulator
neutrino physics

$$\boxed{N = \frac{1}{4\pi i}\,\mathbf{tr}\,[\,\mathbf{K}\oint dl\,\mathbf{H}^{-1}\,\partial_l\,\mathbf{H}]}$$

N=+1 | N=+1 | N=0 | N=+2



$E=cp$ | $E=cp$ | $E^2 = 2c^2p^2 + 4m^2$ | $E^2 = (p^2/\,2m)^2$

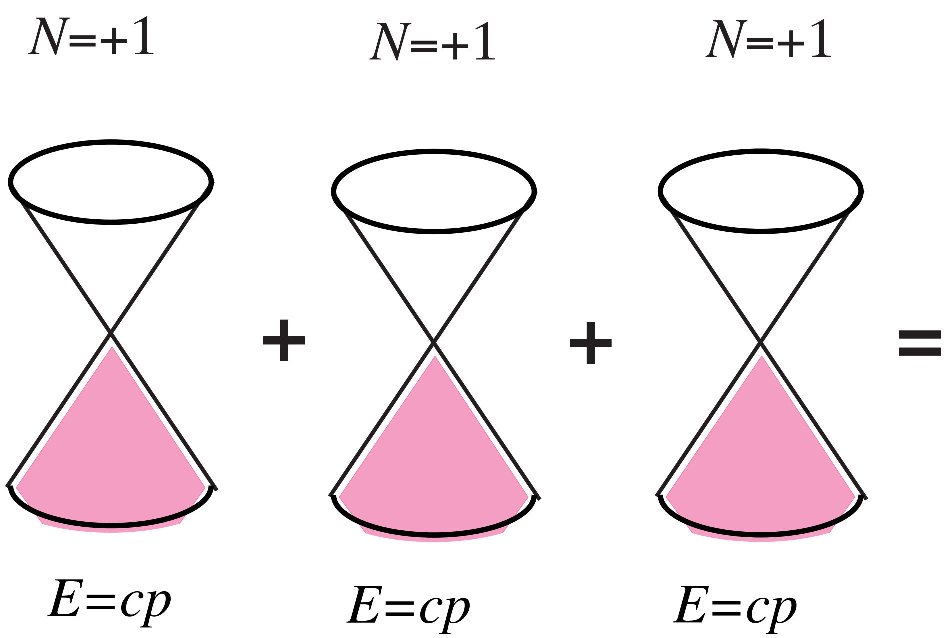
Dirac fermions | massive fermions | massless fermions with quadratic dispersion

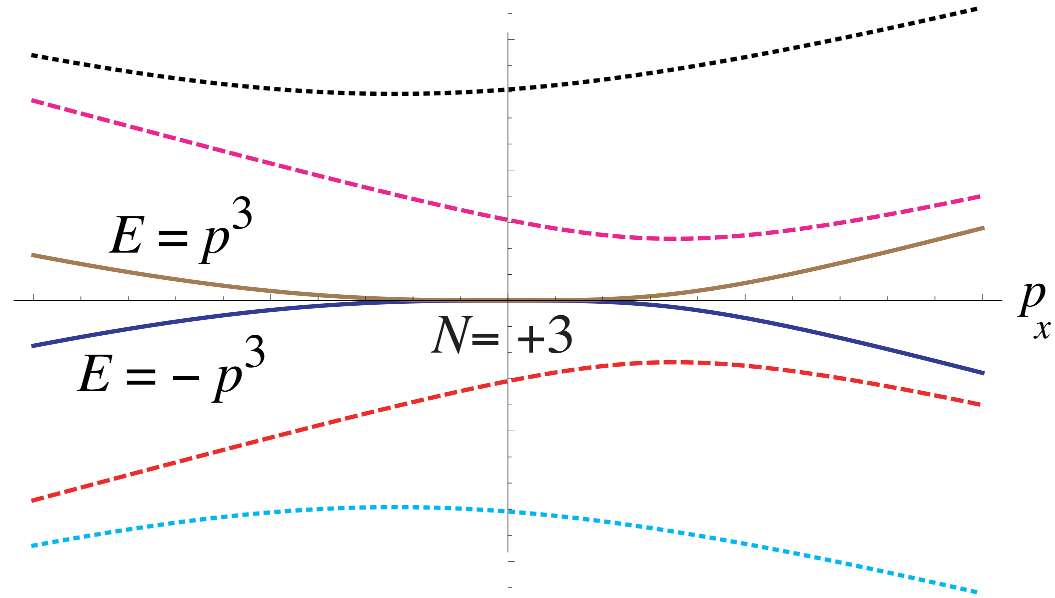
multiple Fermi point

T. Heikkilä & GV arXiv:1010.0393

cubic spectrum in trilayer graphene

$$N = 1 + 1 + 1 = 3$$





multilayered graphene

$$N = 1 + 1 + 1 + \ldots$$

spectrum in the outer layers

$$E = p^N$$

$$E = -\, p^N$$

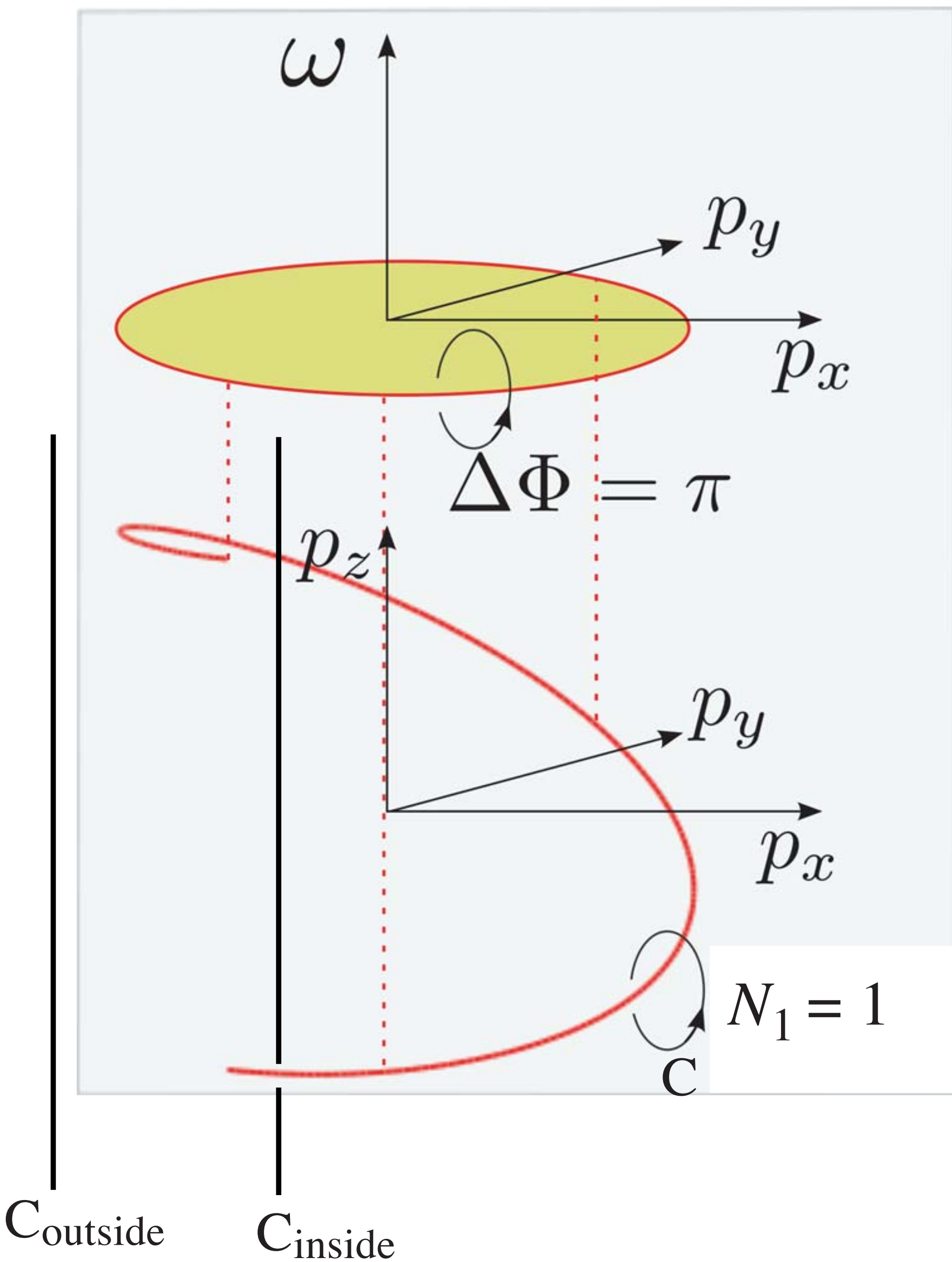
what kind of induced gravity emerges near degenerate Fermi point?

route to topological flat band on the surface of 3D material

Nodal spiral generates flat band on the surface

projection of spiral on the surface determines boundary of flat band

$$\boxed{N_1 = \frac{1}{4\pi i}\mathbf{tr}\left[\mathbf{K}\oint_C dl\, \mathbf{H}^{-1}\,\partial_l\,\mathbf{H}\right]}$$



at each (p_x,p_y) except the boundary of circle one has 1D gapped state (insulator)

$N_{outside} = 0$ trivial 1D insulator

$N_{inside} = 1$ topological 1D insulator

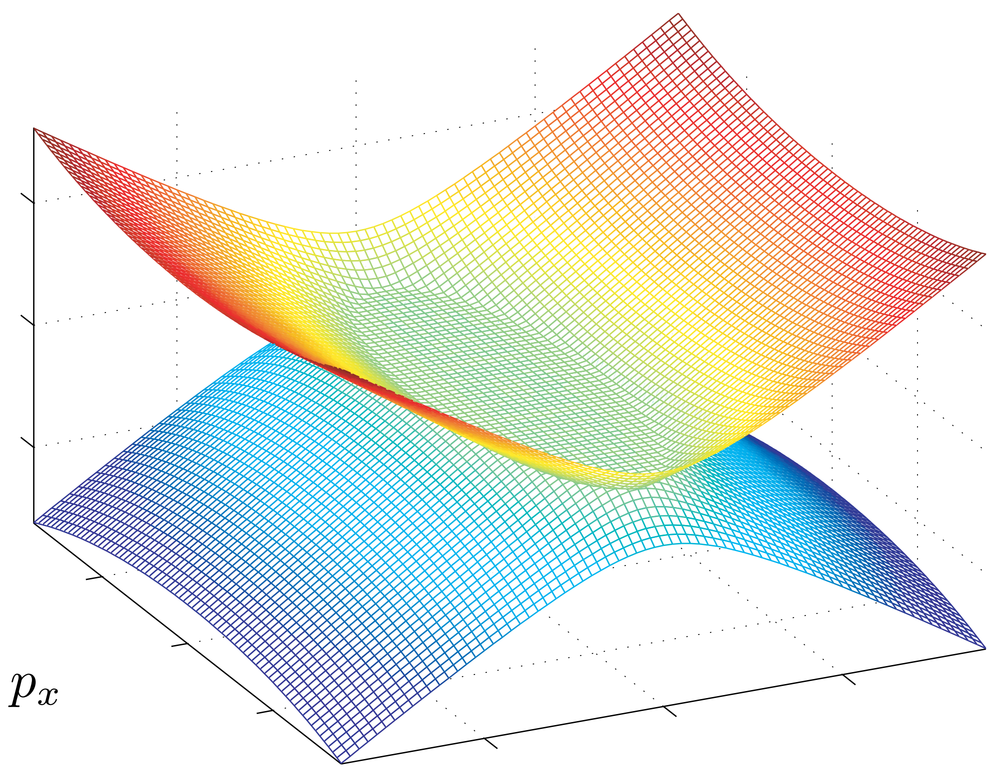
at each (p_x,p_y) inside the circle
one has 1D gapless edge state
this is flat band

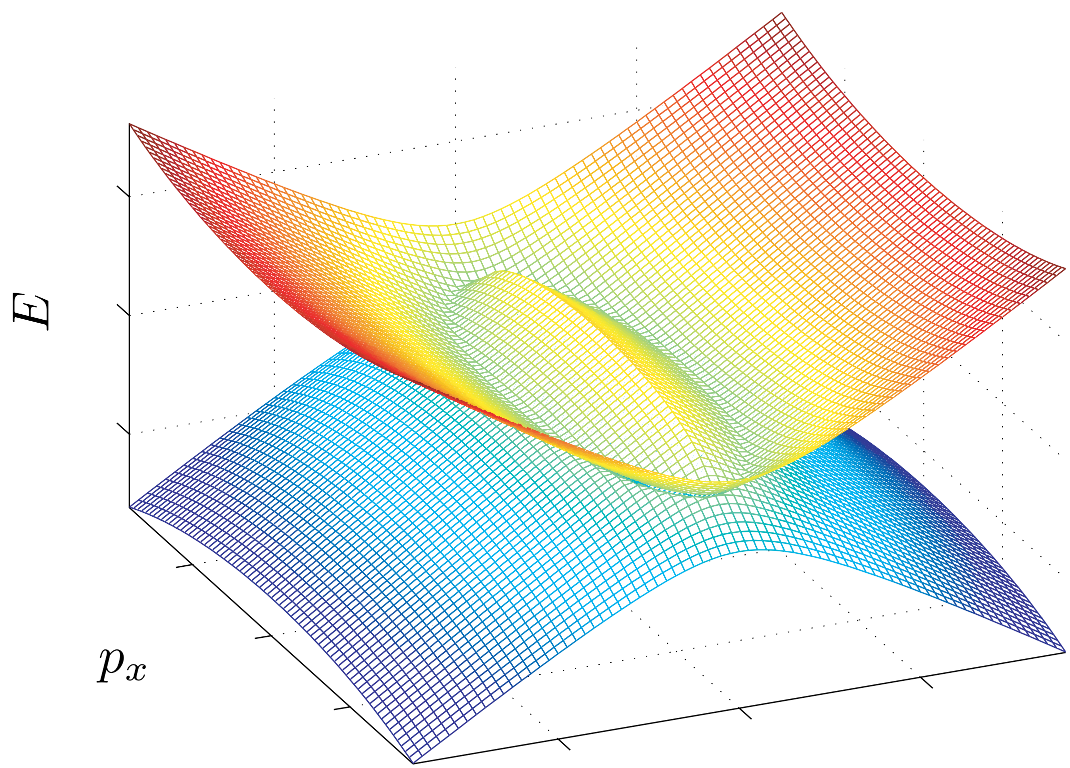
Nodal spiral generates flat band on the surface

projection of nodal spiral on the surface determines boundary of flat band

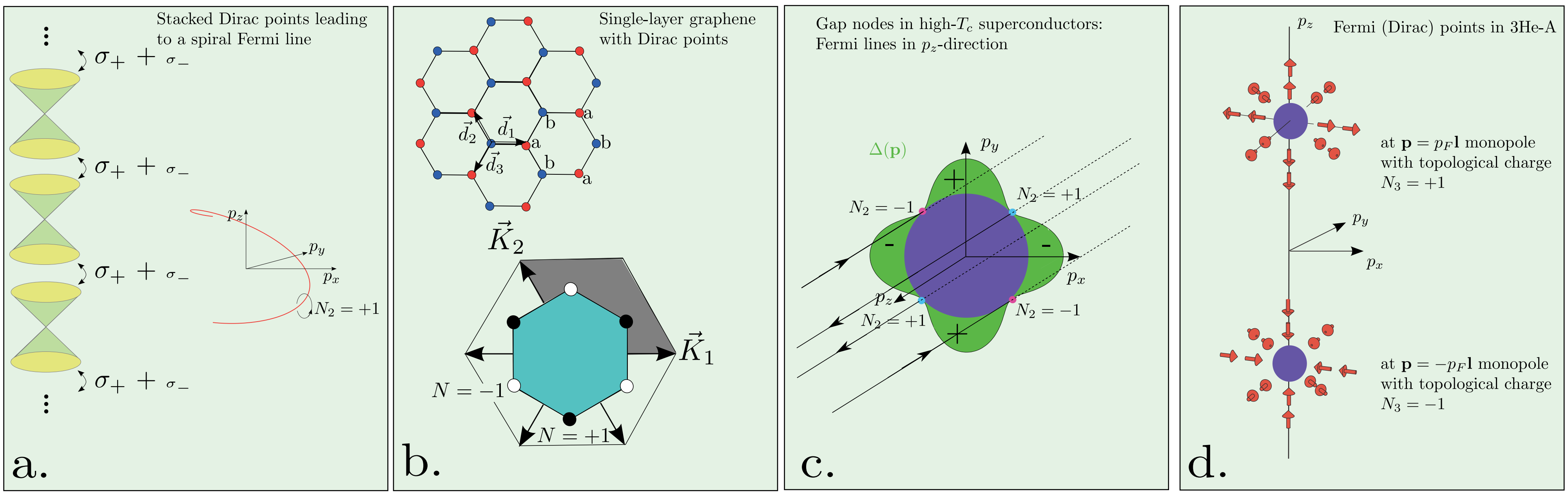
lowest energy states:
surface states form the flat band

energy spectrum in bulk
(projection to p_x , p_y plane)





Gapless topological matter with protected flat band on surface or in vortex core



non-topological flat bands due to interaction

Khodel-Shaginyan fermion condensate

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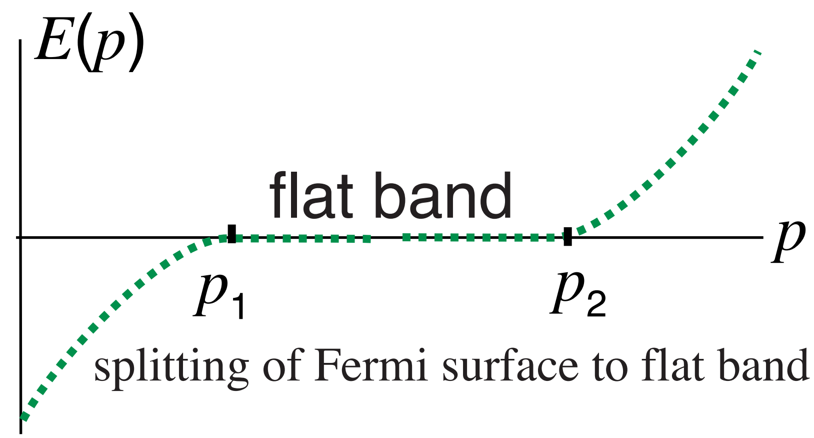
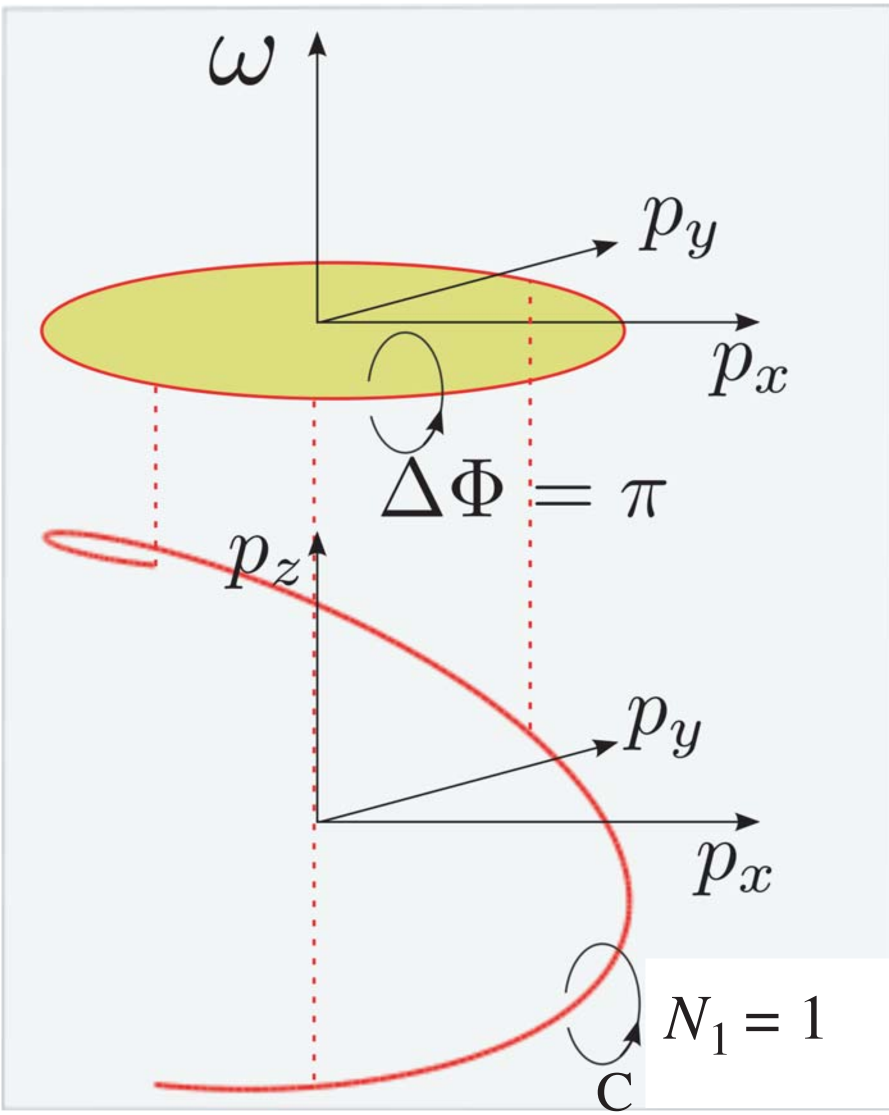


Bulk-surface correspondence:
Flat band on the surface of topological semimetals



flat band on the surface

projection of spiral on the surface
determines boundary of the flat band

topologically protected nodal line
in the form of spiral

$$\boxed{N_1 = \frac{1}{4\pi i} \mathbf{tr} \left[\mathbf{K} \oint_C dl\, \mathbf{H}^{-1} \partial_l \mathbf{H} \right]}$$

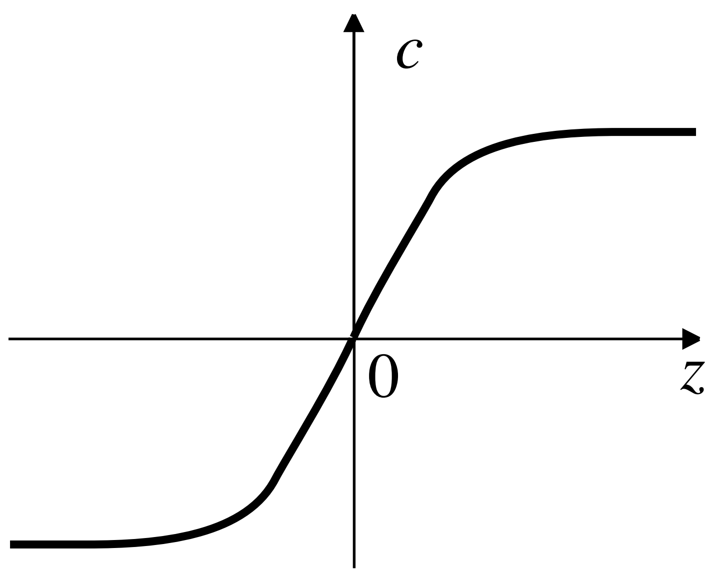
flat band in soliton

$$H = \tau_3 (p_x^{\,2} + p_z^{\,2} - p_F^{\,2})/2m + \tau_1 c(z) p_z$$

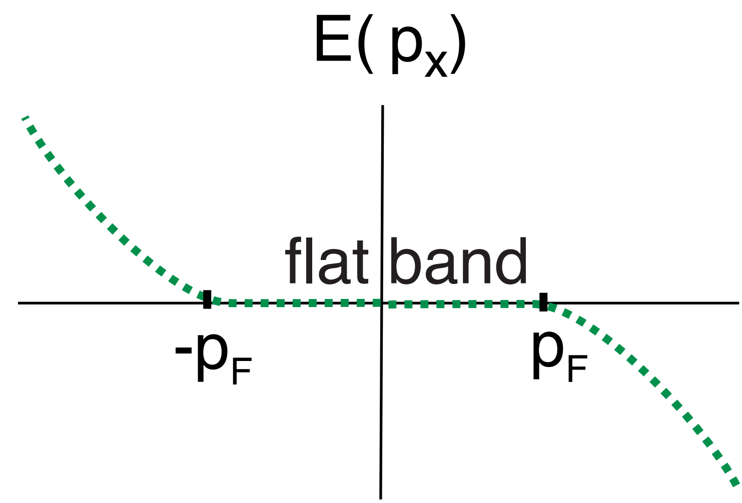
nodes at $p_z = 0$ and $p_x^{\,2} = p_F^{\,2}$

$$\boxed{N = \frac{1}{4\pi i}\mathbf{tr}\left[\mathbf{K}\oint dl\, \mathbf{H}^{-1}\partial_l \mathbf{H}\right]}$$

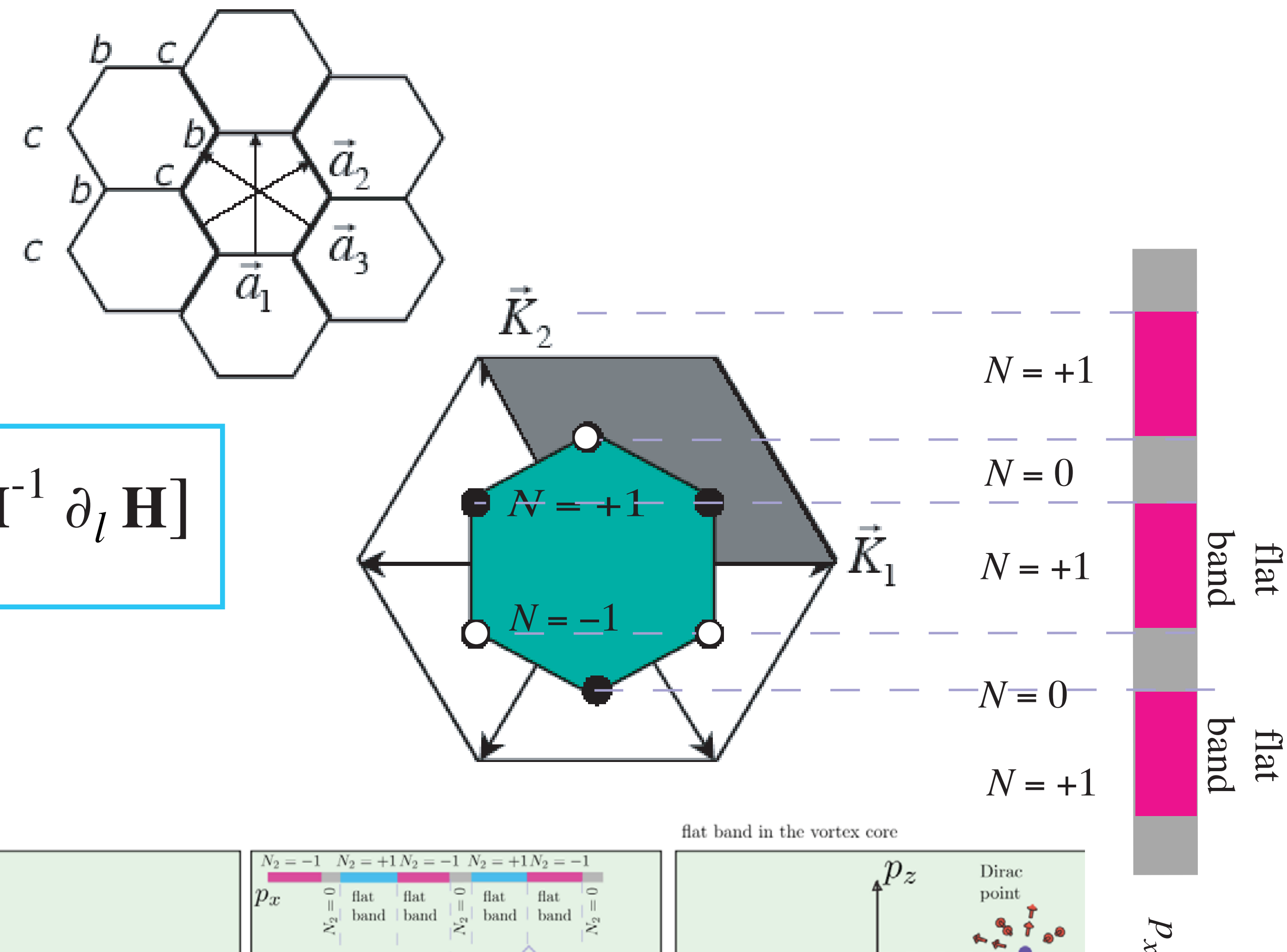
soliton



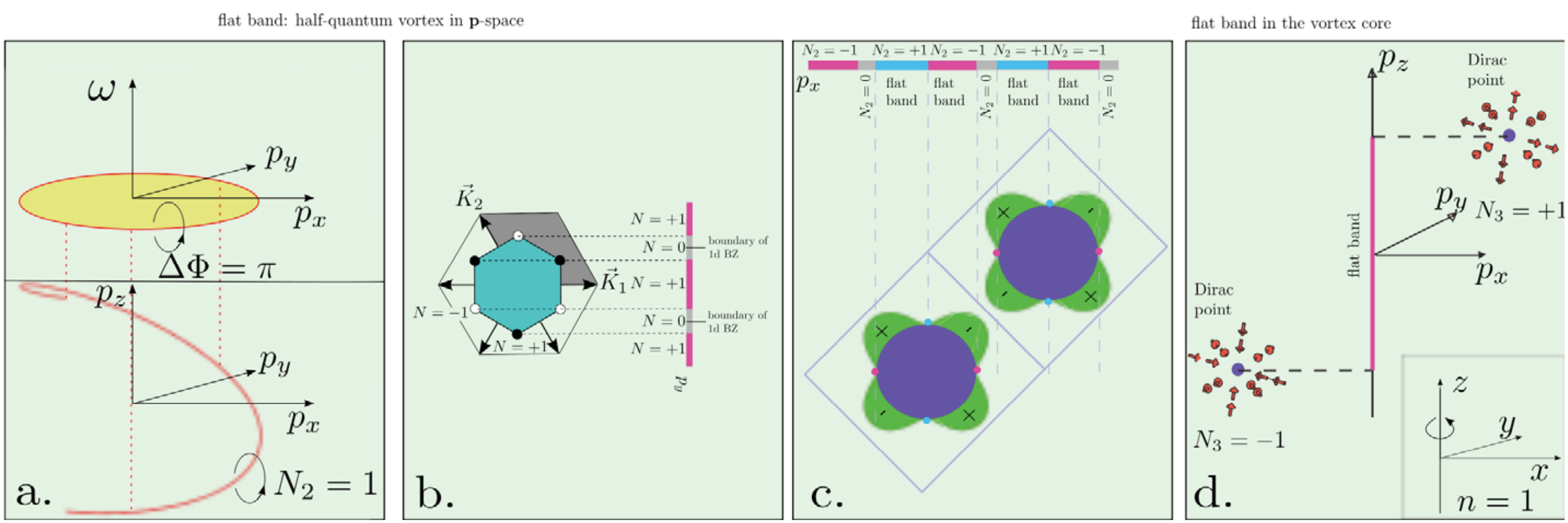
spectrum in soliton



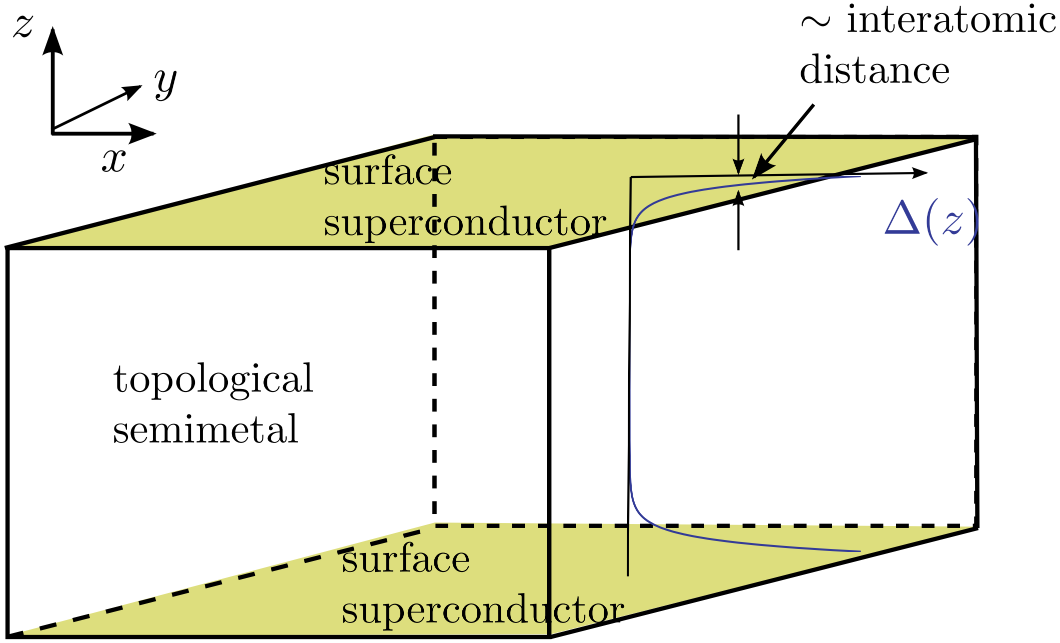
Flat band on the graphene edge

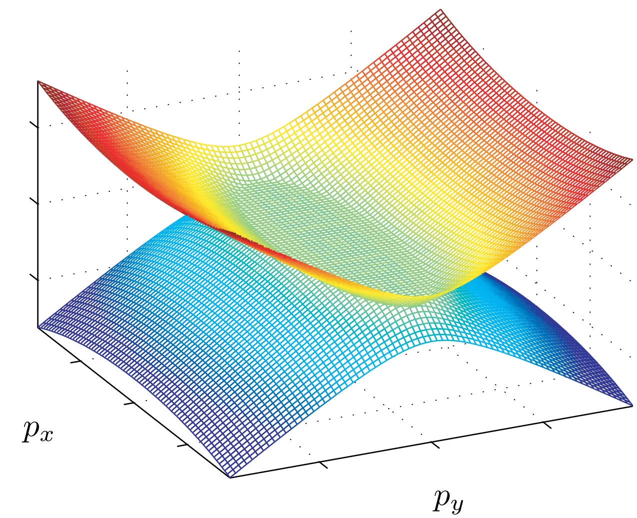


$$N = \frac{1}{4\pi i}\,\mathbf{tr}\left[\,\mathbf{K}\oint dl\,\mathbf{H}^{-1}\,\partial_l\,\mathbf{H}\right]$$



Surface superconductivity in topological semimetals: route to room temperature superconductivity





Extremely high DOS of flat band gives high transition temperature:

normal superconductors: exponentially suppressed transition temperature

$$T_c = T_F \exp\left(-1/g\nu\right)$$

interaction ***DOS***

$$1 = g \int \frac{d^2p}{2h^2} \frac{1}{E(p)}$$

flat band superconductivity: linear dependence of T_c on coupling g

$$T_c \sim g S_{\mathrm{FB}}$$

interaction ***flat band area***

"Recent studies of the correlations between the internal microstructure of the samples and the transport properties suggest that superconductivity might be localized at the interfaces between crystalline graphite regions of different orientations, running parallel to the graphene planes." PRB. 78, 134516 (2008)

PHYSICAL REVIEW B **81**, 184513 (2010)

Stripes of increased diamagnetic susceptibility in underdoped superconducting $Ba(Fe_{1-x}Co_x)_2As_2$ single crystals: Evidence for an enhanced superfluid density at twin boundaries

B. Kalisky,[1,2,*,†] J. R. Kirtley,[1,2,3] J. G. Analytis,[1,2,4] Jiun-Haw Chu,[1,2,4] A. Vailionis,[1,4]
I. R. Fisher,[1,2,4] and K. A. Moler[1,2,4,5,*,‡]

Kathryn Moler:
possible 2D superconductivityof twin boundaries

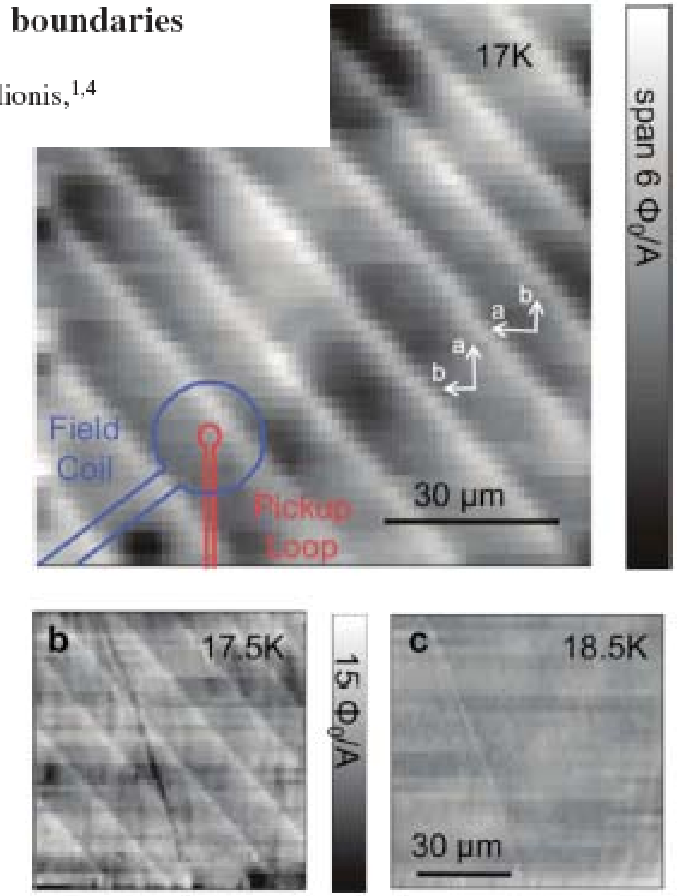
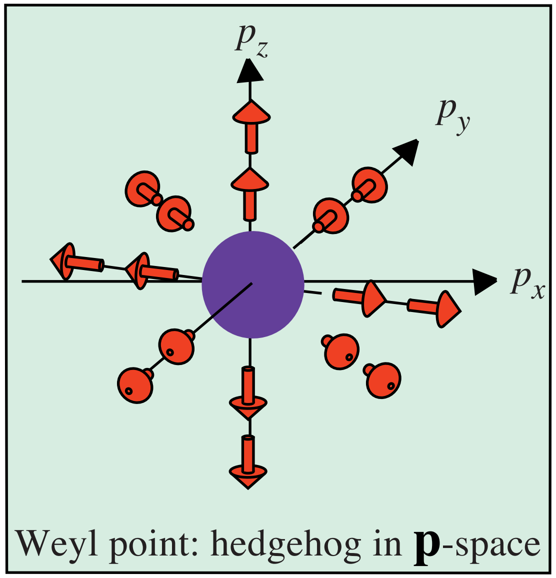


FIG. 1. (Color online) Local susceptibility image in underdoped $Ba(Fe_{1-x}Co_x)_2As_2$, indicating increased diamagnetic shielding on twin boundaries. (a) Local diamagnetic susceptibility, at $T=17$ K, of the *ab* face of sample UD1 ($x=0.051$ and $T_c=18.25$ K), showing stripes of enhanced diamagnetic response (white). In addition there is a mottled background associated with local T_c variations that becomes more pronounced as $T \rightarrow T_c$. Overlay: sketch of the scanning SQUID's sensor. The size of the pickup loop sets the spatial resolution of the susceptibility images. [(b) and (c)] Images of the same region at (b) $T=17.5$ K and (c) at $T=18.5$ K show that the stripes disappear above T_c. A topographic feature (scratch) appears in (b) and (c).

Weyl point



$$N_3 = \frac{1}{8\pi} e_{ijk} \int dS^k \; \hat{\mathbf{g}} \cdot (\partial_{p_i} \hat{\mathbf{g}} \times \partial_{p_j} \hat{\mathbf{g}}) = 1$$

over 2D surface S
in 3D momentum space

primary object:
tetrad
$e_a{}^\mu$

relativistic quantum fields & gravity emerging near Weyl point

Atiyah-Bott-Shapiro construction:
linear expansion near Weyl point in terms of Dirac Γ-matrices

secondary object:
metric
$g^{\mu\nu} = \eta^{ab} e_a{}^\mu e_b{}^\nu$

emergent relativity $\boxed{H = e_a{}^k \Gamma^a \cdot (p_k - p_k^0)}$

$$\boxed{g^{\mu\nu}(p_\mu - eA_\mu - e\tau \cdot \mathbf{W}_\mu)(p_\nu - eA_\nu - e\tau \cdot \mathbf{W}_\nu) = 0}$$

effective metric:
emergent gravity

effective
electromagnetic
field

effective
$SU(2)$ gauge
field

effective
electric charge
$e = +1$ or -1

effective
isotopic spin

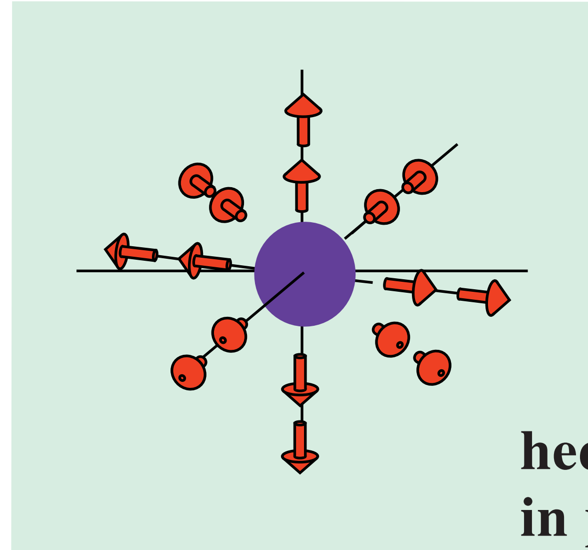
all ingredients of Standard Model :
chiral fermions & gauge fields
emerge in low-energy corner
together with spin,
Dirac Γ−matrices,
gravity & physical laws:
Lorentz & gauge invariance,
equivalence principle, etc

From Weyl point to quantum Hall topological insulators

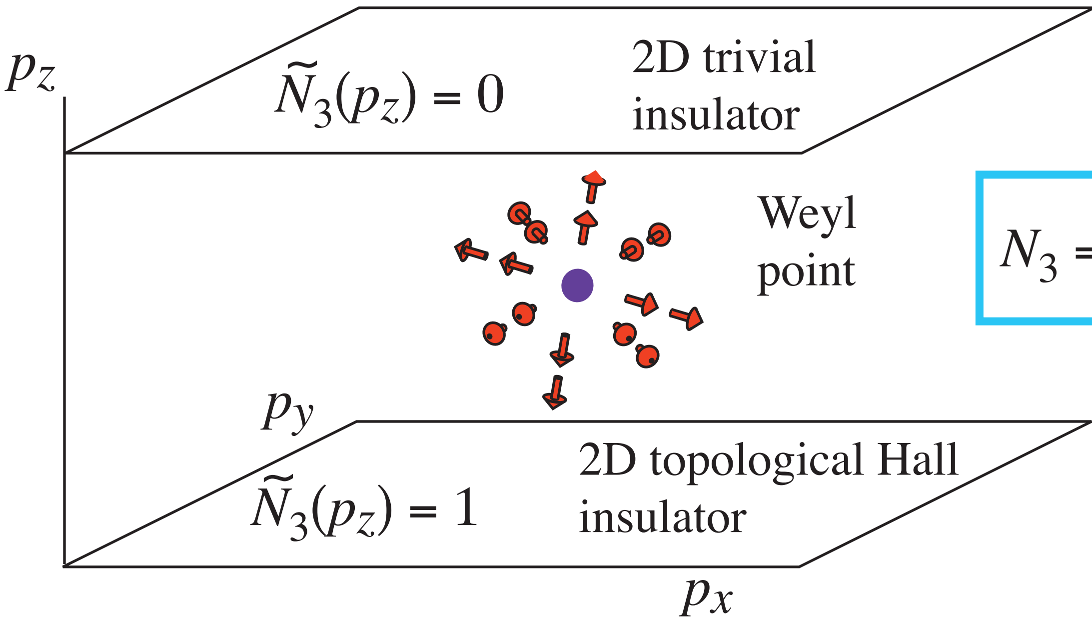
$$N_3 = \frac{1}{8\pi} e_{ijk} \int dS^k \; \hat{\mathbf{g}} \cdot (\partial_{p_i} \hat{\mathbf{g}} \times \partial_{p_j} \hat{\mathbf{g}})$$

over 2D surface S
in 3D momentum space

top. invariant for Weyl point in 3+1 system



hedgehog in p-space



$$N_3 = \tilde{N}_3(p_z < p_0) - \tilde{N}_3(p_z > p_0)$$

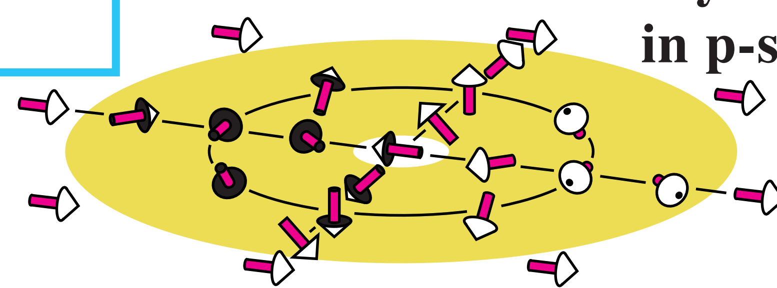
at each p_z one has 2D insulator or fully gapped 2D superfluid

$$\tilde{N}_3(p_z) = \frac{1}{4\pi} \int dp_x dp_y \; \hat{\mathbf{g}} \cdot (\partial_{p_x} \hat{\mathbf{g}} \times \partial_{p_y} \hat{\mathbf{g}})$$

over the whole 2D momentum space
or over 2D Brillouin zone

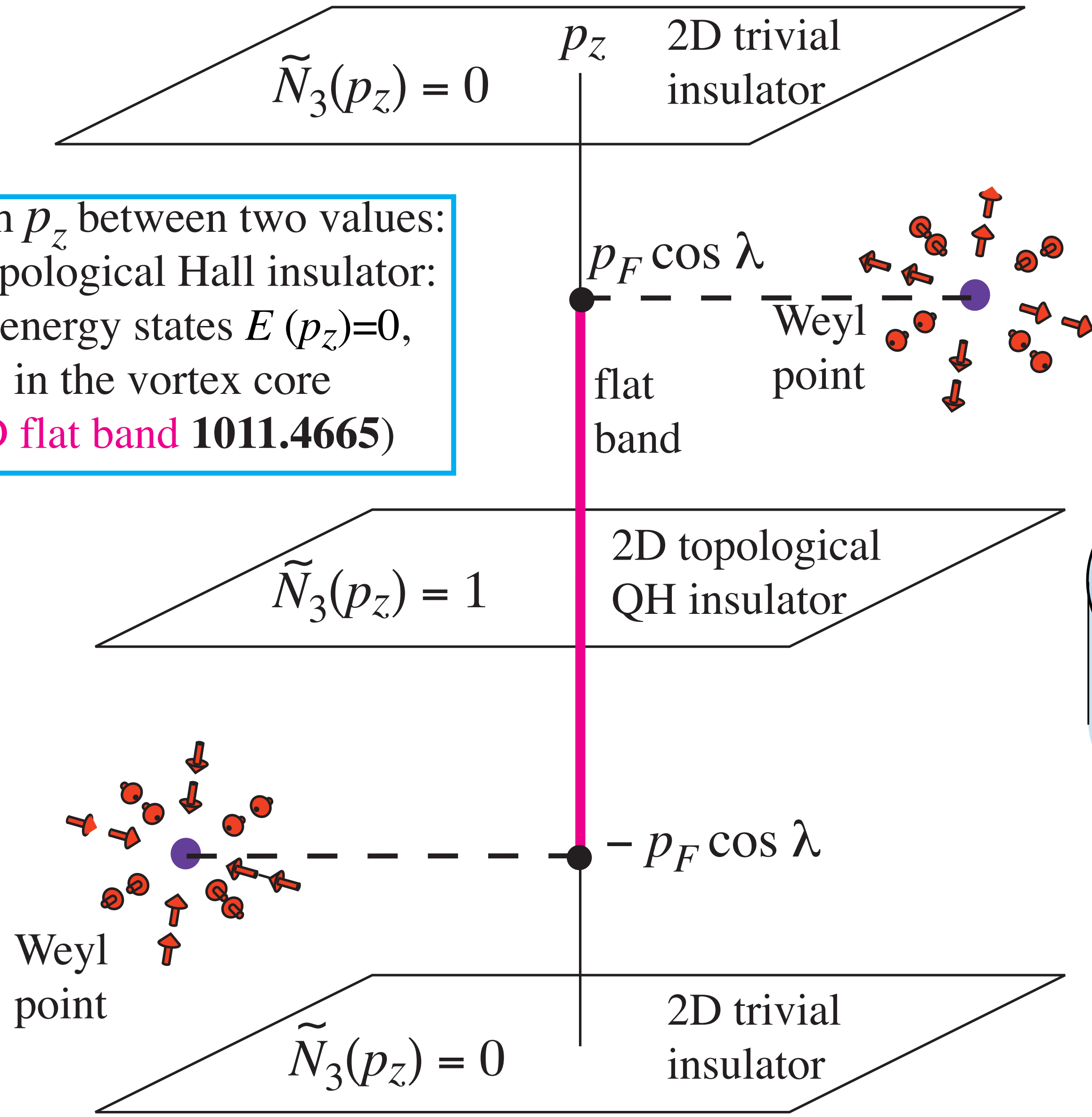
top. invariant for fully gapped 2+1 system

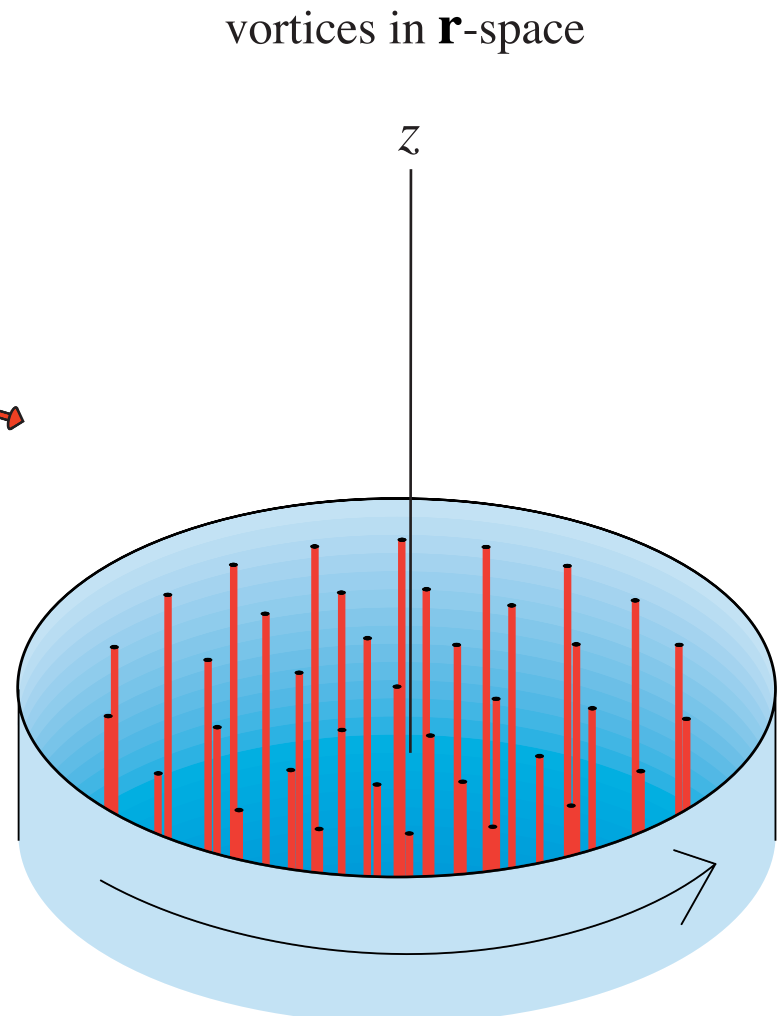
skyrmion in p-space



3D matter with Weyl points:
Topologically protected flat band in vortex core

at each p_z between two values:
2D topological Hall insulator:
zero energy states $E\,(p_z)$=0,
in the vortex core
(1D flat band **1011.4665**)

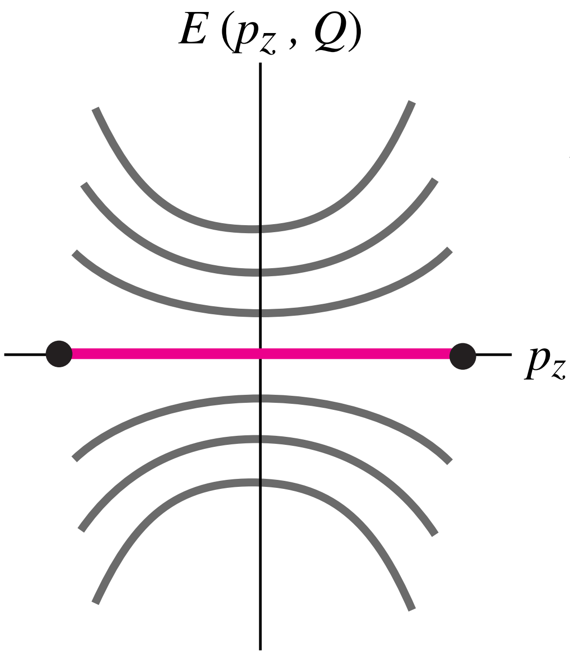




$$\tilde{N}_3(p_z) = \frac{1}{4\pi^2}\,\mathbf{tr}\int dp_x\, dp_y d\omega\; \mathbf{G}\,\partial_\omega\, \mathbf{G}^{-1}\, \mathbf{G}\,\partial_{p_x}\, \mathbf{G}^{-1}\mathbf{G}\,\partial_{p_y}\, \mathbf{G}^{-1}$$

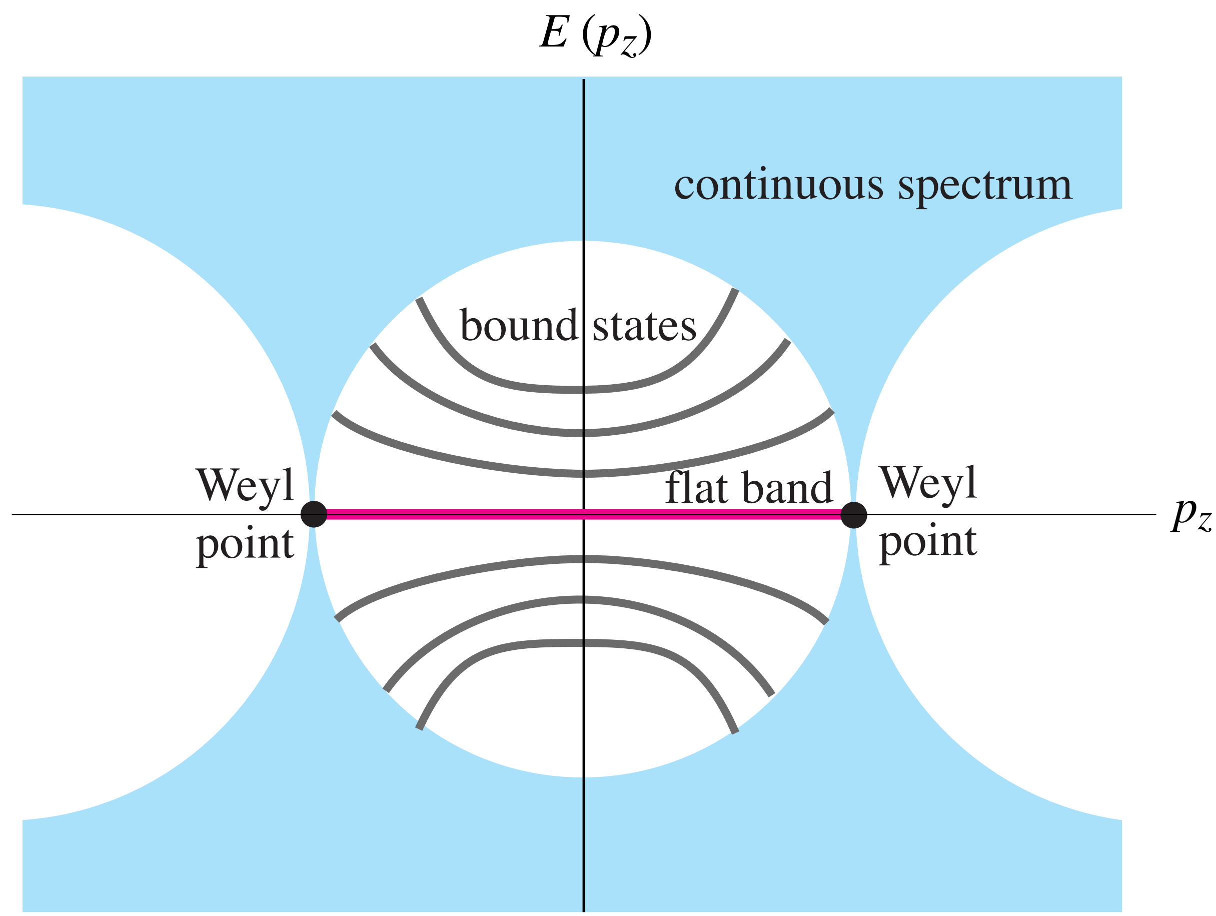
GV & Yakovenko
(1989)

Topologically protected flat band in vortex core of superfluids with Weyl points



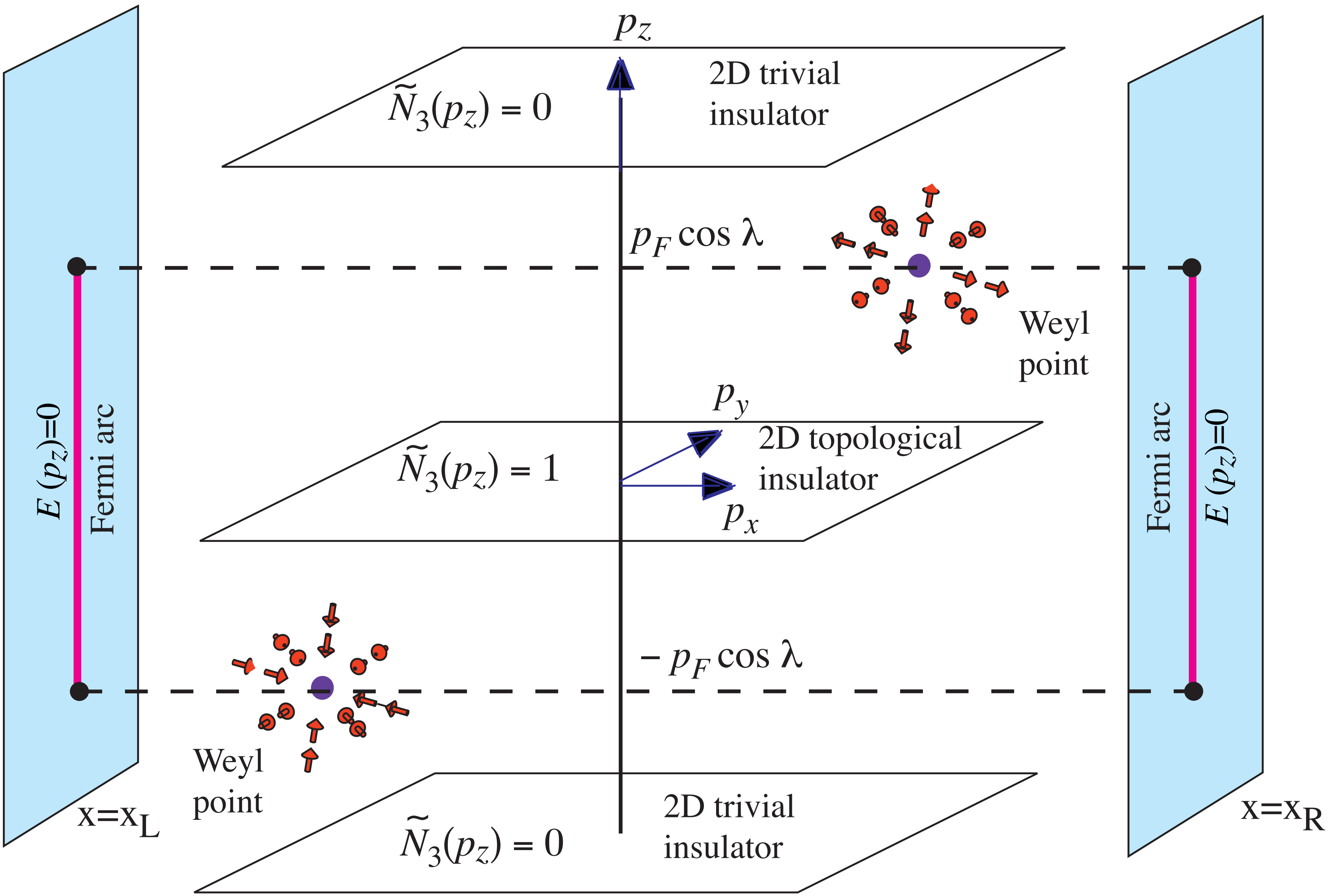
flat band
in spectrum of fermions
bound to core of 3He-A vortex
(Kopnin-Salomaa 1991)

flat band of bound states
terminates on zeroes
of continuous spectrum
(i.e on Weyl points)

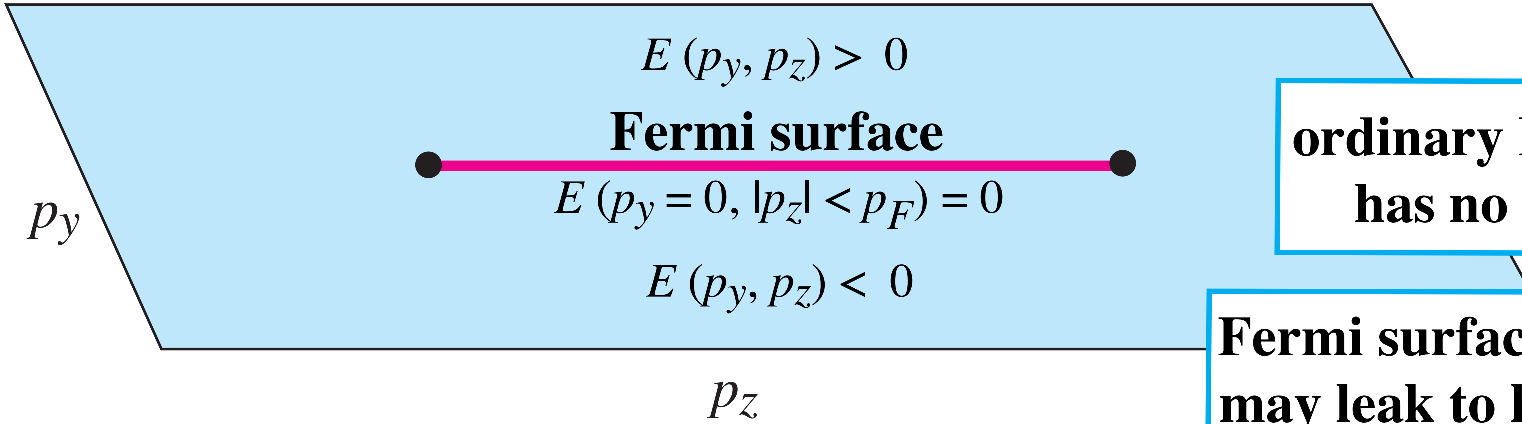


3He-A with Weyl points: Topologically protected Fermi arc on the surface

for each $|p_z| < p_F \cos \lambda$
one has 2D topological Hall insulator with
zero energy edge states $E(p_z)=0$
(Dirac valley or Fermi arc **PRB 094510,PRB 205101**)



Fermi arc:
Fermi surface separates positive and negative energies, but has boundaries

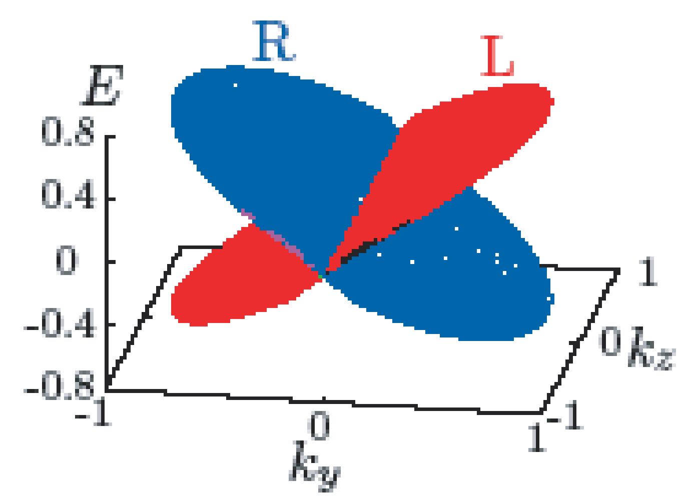


ordinary Fermi surface has no boundaries

Fermi surface of bound states may leak to higher dimension

Fermi surface of localized states is terminated by projections of Weyl points when localized states merge with continuous spectrum

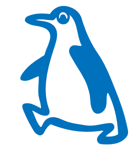
L spectrum of edge states on left wall



PRB 094510

R spectrum of edge states on right wall

Conclusion: **p-space topology** determines fundamental many-body quantum phenomena



universality classes of quantum vacua

effective field theories in these quantum vacua (emergent gravity & QED)

topological quantum phase transitions (Lifshitz, plateau, etc.)

quantization of Hall and spin-Hall conductivity

topological Chern-Simons & Wess-Zumino terms

quantum statistics of topological objects

bulk-surface & bulk-vortex correspondence

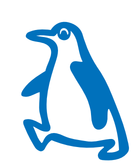
exotic fermions: Majorana fermions; flat band; Fermi arc; cubic, quartic etc. dispersion

chiral anomaly & vortex dynamics, etc.

flat band & room-temperature superconductivity

superfuid phases ^{3}He serve as primer for topological matter: quantum vacuum of Standard Model, topological superconductors, topological insulators, topological semimetals, etc.

we need: low T, high H, miniaturization, atomically smooth surface, nano-detectors, ...
fabrication of samples for room-temperature superconductivity



unity of physics

^{3}He Grand Unification

Emergence & effective theories
Weyl, Majorana & Dirac fermions
Vacuum polarization, screening - antiscreening, running coupling
Symmetry breaking (anisotropy of vacuum)
Parity violation -- chiral fermions
Vacuum instability in strong fields, pair production
Casimir force, quantum friction
Fermionic charge of vacuum
Higgs fields & gauge bosons
Momentum-space topology
Hierarchy problem, Supersymmetry

Torsion & spinning strings, torsion instanton
Fermion zero modes on strings & walls
Antigravitating (negative-mass) string
Gravitational Aharonov-Bohm effect
Domain wall terminating on string
String terminating on domain wall
Monopoles on string & Boojums
Witten superconducting string
Soft core string, Q-balls
Z &W strings
skyrmions
Alice string
Pion string

cosmic strings

physical vacuum

Neutrino oscillations
Chiral anomaly & axions
Spin & isospin, String theory
CPT-violation, GUT

Gap nodes
Low -T scaling
mixed state
Broken time reversal
1/2-vortex, vortex dynamics

Kibble mechanism
Dark matter detector
Primordial magnetic field
Baryogenesis by textures
& strings
Inflation
Branes
matter
creation

Cosmological &
Newton constants
dark energy
dark matter
Effective gravity
Bi-metric &
conformal gravity
Graviton, dilaton
Spin connection
Rotating vacuum
Vacuum dynamics
conformal anomaly
ergoregion, event horizon
Hawking & Unruh effects
black hole entropy
Vacuum instability

cosmology

High Energy Physics

high-T & chiral super-conductivity

vacuum gravity

Gravity

^{3}He

Condensed Matter

1D, 2D systems

topological insulators, semimetals

Films: FQHE,
Statistics & charge of
skyrmions & vortices
Edge states;
flat band, Fermi arc
spintronics
1D fermions in vortex core
Critical fluctuations
Mixture of condensates
Vector & spinor condensates
BEC of quasipartcles,
magnon BEC & laser
meron, skyrmion, 1/2 vortex

black holes

QCD

Plasma Physics

Phenomenology

BEC

neutron stars

quark matter

nuclear physics

phase transitions

disorder

hydrodynamics

General; relativistic;
spin superfluidity
multi-fluid
rotating superfluid
Shear flow instability
Magnetohydrodynamic
Turbulence of vortex lines
propagating vortex front
velocity independent
Reynolds number

Superfluidity of neutron star
vortices, glitches
shear flow instability

Quark condensate
Nambu--Jona-Lasinio
Vaks--Larkin
Color superfluidity
Savvidi vacuum
Quark confinement, QCD cosmology
Intrinsic orbital momentum of quark matter

Nuclei vs
^{3}He droplet
Shell model
Pair-correlations
Collective modes

quantum phase transitions
& momentum-space topology

Relativistic plasma
Photon mass
Vortex Coulomb
plasma

random
anisotropy
Larkin- Imry-Ma
classes of random
matrices